Beyond tree-shaped credal sum-product networks

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1 Sum-product networks

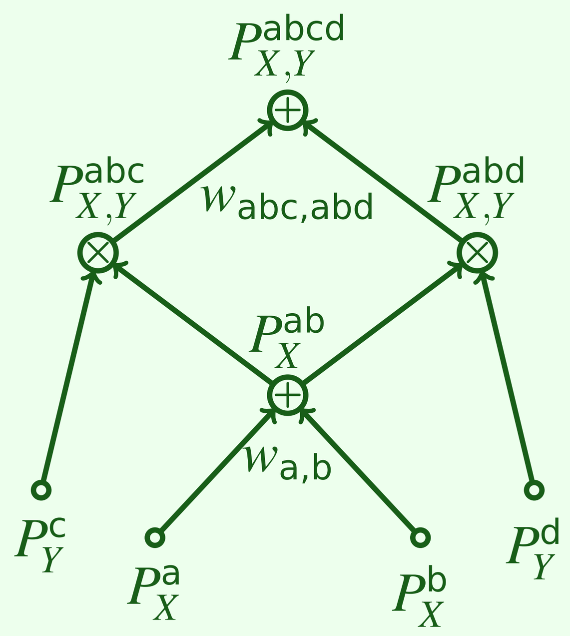
A **sum-product network (SPN)** models a joint probability distribution.

Context

- AI and ML applications with a large number of random variables
- Requirement to efficiently calculate probabilities and expectations
- Graphical representations of joint probability distributions have intuition-building appeal

Definition of SPNs

- Built from typically univariate **'leaf' distributions** (P_X^{b}, P_Y^{d}, ...)
- Combined by repeated application of
  - independent **'product' operations** ($\otimes$ nodes); its factors' variables must be distinct (**'decomposability'**)
  - **'sum' operations** ($\oplus$ nodes), which are convex mixtures characterized by **weight vectors** ($w_{\mathrm{a,b}}$, $w_{\mathrm{abc,abd}}$); its terms' variables must be identical (**'smoothness'**)
- Resulting in a **joint 'root' distribution**



Properties of SPNs

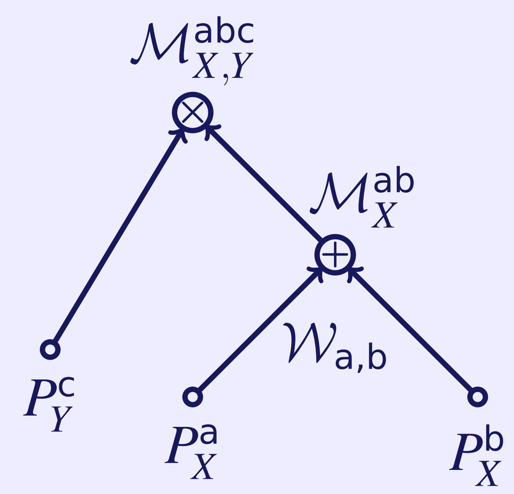
- Efficient computation of expectations of **'query' functions that factorize** over the random variables: $f(X,Y) = f(X)f(Y)$
- An SPN's underlying structure is an **acyclic directed graph (DAG)**

2 Credal sum-product networks

A **credal sum-product network (CSPN)** models a joint credal set.

Definition of CSPNs

- Use sensitivity analysis-style **complete product** for independent product operations
- Add imprecision by using **sets of weight vectors** ($\mathcal{W}_{\mathrm{a,b}}$) in sum nodes, resulting in credal sets ($\mathcal{M}_X^{\mathrm{ab}}$, $\mathcal{M}_{X,Y}^{\mathrm{abc}}$)



Limitations of the state of the art (Mauá et al., 2018)

Efficient computation of lower and upper expectations demonstrated only for

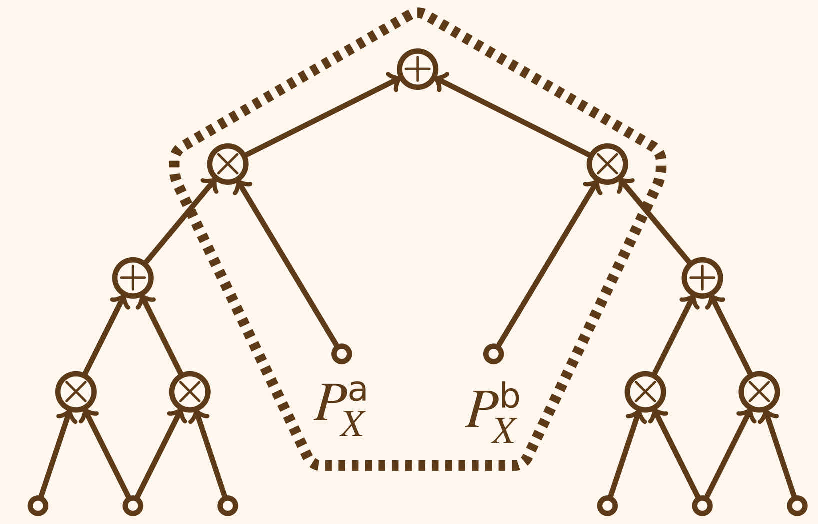
- CSPNs with **tree-shaped graphs**, together with
- **indicator-difference 'focused' query functions**: factorizing functions where
  - all but one factor is an indicator (e.g., $f(Y) = I_{\hat{y}}(Y)$)
  - the non-indicator 'focus factor' is a difference of indicators (e.g., $f(X) = I_{\hat{x}}(X) - I_{\check{x}}(X)$); it can be any function for general focused query functions

3 Theoretical results

We can **relax the restriction to focused queries and trees** if the combination of these relaxations is compatible with efficient inference of lower and upper expectations.

Our contribution: cases beyond the state of the art

1. Query: general factorizing function
   Graph: tree
2. Query: factorizing function with all factors non-negative
   Graph: DAG
3. Query: general focused function
   Graph: DAG for which no cycle edge is part of the paths from root to focus variable leafs (see figure)

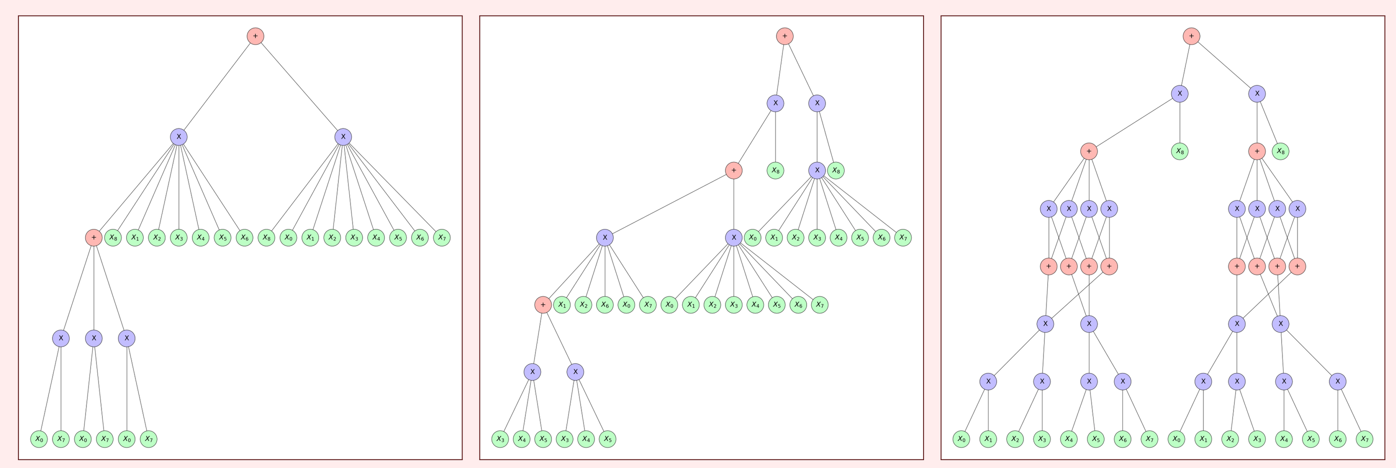


4 Experimental results

- An **implementation for non-tree-shaped CSPNs** has been made
- Credal classification used as the test application
- **DAG-shaped CSPNs sometimes perform better**, for some datasets and parameters

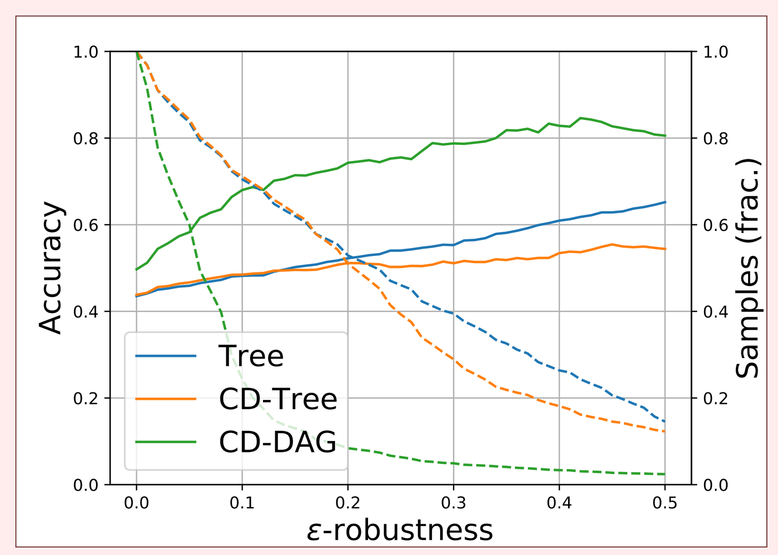
Graph shapes used

- arbitrary tree (left example)
- class-discriminative tree (middle); focus (class) variable leafs below highest product nodes
- class-discriminative DAG (right)

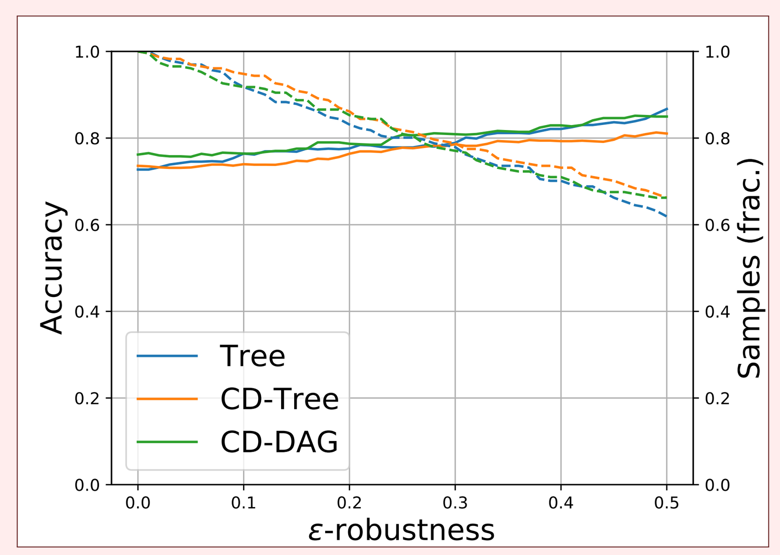


Comparison between shapes, datasets, and evidence types

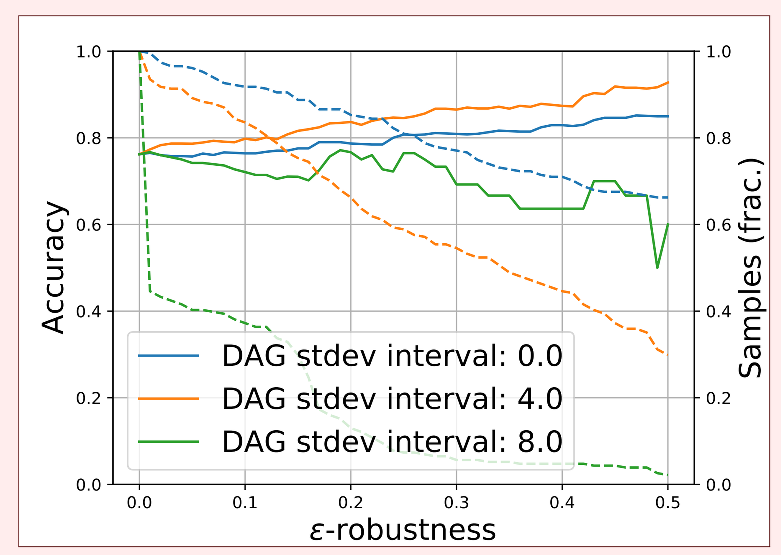
- Plots below are a **function of imprecision** (ε-contamination of weight vectors)
- Show **singleton classification accuracy** (full lines) **and fraction** (dashed lines)



'Gesture' dataset: impact of shape



'Diabetes' dataset: impact of shape



'Diabetes' dataset: impact of soft evidence width