This poster introduces a procedure for eliciting coherent sets of acceptable gambles on three-outcome possibility spaces. We also discuss a real-life experiment conducted as an exploratory test of this elicitation interface; it was organized around the 2014 FIFA World Cup.

Because I’m inside a yellow box, I’m running example or some other illustration!

- **Introducing Sets of Acceptable Gambles**

  **The CWI World Cup Competition**

- **Essential Concepts**

  - **Possibility space** \( \Omega \), finite set of possible experimental outcomes.
  - **Gamble Space Representation**
  - **Coherence axioms**
  - **Assessment**
  - **Acceptable gamble**
  - **Possibility space** \( \Omega \)
  - **Natural extension**
  - **Reference value**
  - **Surface to project**

  - **An example:** \( g = 4I - 4W = (-1, 0, 4) \), with \( I \) indicator function notation.

- **Acceptable gamble**

  - An elicitor finds a gamble \( g \) acceptable if she is committed to receiving the payoff \( g(\omega) \) once the actual outcome \( \omega \in \Omega \) is determined.

- **Assessment** \( A \), a set of limited gambles assessed to be acceptable:

  \[
  A = \{ (6I - 1, 6W - 1, 4I - 4W) \}.
  \]

- **Coherence axioms**

  - A coherent set of acceptable gambles \( D \) should satisfy:
    - Avoiding Sure Loss: \( g < 0 \Rightarrow g \in D \).
    - Addition: \( g, h \in D \Rightarrow g + h \in D \).
    - Positive Homogeneity: \( g \geq 0 \Rightarrow \lambda g \in D \).
    - Accepting Partial Gains: \( g \geq 0 \Rightarrow g \in D \).

  \( D \) is a convex cone that includes the positive orthant and does not intersect the negative one.

- **Natural extension**

  - The smallest set of acceptable gambles that includes an assessment \( A \),
  - \( D = \{ f + \sum \lambda h : f \geq 0, \lambda \geq 0 \} \).

- **Intersection of \( D \) with the plane of gambles whose payoffs sum to one:

  \[
  I_{ND} - I_{W} = \frac{1}{4}
  \]

  (Dashed triangle delimits positive octant.)

- **Lower expectation or Preview**

  - The supremum acceptable buying price for the gamble \( h, \)
  - \( E(h) = \sup \{ a \in \mathbb{R} : a - \alpha \geq 0 \} \).

- **Credal set**

  - A convex subset of the probability simplex,
  - \( M = \{ P : E \subseteq F \} \).

- **Gamble Space Representation**

  - **Problem** Not all coherent sets of acceptable gambles can be (comparably) depicted by their intersection with a plane, as was done above.

- **Considerations**

  - Representation on a two-dimensional surface is possible by Positive Homogeneity.

  \[
  \sum_{o} g(o) = 0 \text{ for } g \in D,
  \]

  \[
  \sum_{o} P(o) g(o) = 0 \text{ for } P \in M.
  \]

- **An instance of the experiment’s interface, including an assigned gamble:**

- **Implementation**

  - **Discretization**
    - Computing natural extension Responsively.
    - Show gamble values on hover, without a distracting number of significant digits.

- **Results**

  - **Match assessments** 194 in total.
  - **Completeness**
    - Proportion of gambles being acceptable or rejected:
    - A good 20% of assessments were complete.
  - **Rerepresentation of**
    - A set of gambles summing to the zero gamble,
    - A pair of opposite ‘simple’ gambles,
    - A pair of opposite ‘simple’ gambles, with equal nonnegative lower expectation.

- **Our fair bets**

  - Between all in a pool of participants:
    - A set of gambles summing to the zero gamble,
    - With equal nonnegative lower expectation,
    - Maximizing the sum of lower expectations (participants could be excluded from the bet).

  - (Involves a mixed-integer linear program.)

- **Walley’s fair bets**

  - Between a pair of participants:
    - A pair of opposite ‘simple’ gambles,
    - With equal nonnegative lower expectation.

- **Selecting dots per assessment**

  - **A participant’s played-match list at the end of the competition:**
    - 17 academic participants; 36 matches
    - 17 academic participants; 36 matches

- **Selected gambles distribution**

  - Primarily gambles on the axes and contingent gambles were chosen, but not overwhelmingly so.

- **Relative gamble selection frequency** (\( \langle \text{dot area} \rangle \):

- **The Experiment**

  - **1982 World Cup** (Walley’s experiment)
    - Eliciting lower and upper probabilities
    - Pen & paper interface (?)
    - 17 academic participants; 36 matches
    - Assessments evaluated using the 6000+ possible pairwise ‘fair’ bets between them

- **2014 World Cup** (Our experiment)

  - Eliciting acceptable gambles
  - On-line point-and-click interface ensuring coherence
  - 80 mostly academic participants; 32 matches
  - Assessments used in a betting pool; 100 ‘fair’ gambles assigned in total

- **Conclusions**

  - When given the option, people provide imprecise assessments.

  - Credal sets for the final match, GER-ARG:
    - The labeled simplex on the left contains the assessment shown earlier for this match.

  - From participant feedback, we learned that the interface needs to be easier to understand.
  - Often, many participants, mostly with relatively imprecise assessments, were excluded from bets. To improve feedback to users, the gamble assignment algorithm should be extended to be more inclusive.