Extra exercise: Inference for exponential sampling

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July 21, 2014

1 Representing the data

Repeat the above analysis for the following sequence of samples:

2.8 8.2 6.2 1.8 9.8 1.4 0.81 0.20 0.24 0.36.

Mean is 3.2.

0.20 0.24 0.36 0.81 1.4 1.8 2.8 6.2 8.2 9.8.

2 Bringing in the sampling model

As a sampling model, you may assume an exponential distribution with probability density function

$$p_\beta(z) := \beta \exp(-\beta z).$$

defined for $z \geq 0$, with $\beta > 0$ the rate parameter, which is also the inverse of the mean.

Likelihood:

$$L_x(\beta) := \prod_{i=1}^{n} L_{x_i}(\beta) = \beta^n \exp(-\beta n \bar{x}).$$

The maximum likelihood estimate of β is equal to the inverse of the sample mean $\bar{x}$, which is again a sufficient statistic. α-Confidence interval for β (needs to be checked!):

$$[\frac{\chi^2_{\frac{1-\alpha}{2},2n}}{2n}\frac{1}{\bar{x}}, \frac{\chi^2_{\frac{1+\alpha}{2},2n}}{2n}\frac{1}{\bar{x}}]$$

3 Bringing in the prior

The conjugate distribution for exponential sampling is the Gamma distribution, with density

$$p_{\tilde{s},\tilde{t}}(\beta) := \frac{(\tilde{s}\tilde{t})^{\tilde{s}+1}}{\Gamma(\tilde{s}+1)}\beta^{\tilde{s}} \exp(-\tilde{s}\tilde{t}\beta),$$

with parameters $\tilde{s} > 0$ and $\tilde{t} > 0$—the mean.

4 From parametric to predictive inference

The predictive probability mass function's expression is:

$$p_{\tilde{s},\tilde{t}}(z) := \frac{\tilde{s}+1}{\tilde{s}\tilde{t}}\left(\frac{\tilde{s}\tilde{t}}{z+\tilde{s}\tilde{t}}\right)^{\tilde{s}+2}.$$