MODELING RISK & UNCERTAINTY USING ACCEPT & REJECT STATEMENTS

Erik Quaeghebeur
Centrum Wiskunde & Informatica, Amsterdam

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Risk and preference is often modeled using linear previsions and linear orders. Some more expressive models use sets of probabilities, lower previsions, or partial orders (see, e.g., Seidenfeld et al., 1990; Walley, 2000) and can also deal with uncertainty. In the discussion of these more expressive models, alternative representations in terms of sets of so-called acceptable, favorable, or desirable gambles appear (cf. Seidenfeld et al., 1990; Walley, 2000). Such 'sets of gambles'-based models are attractive because of their geometric nature.

We generalize these 'sets of gambles'-based models by considering a *pair* of sets, one with accepted gambles and one with rejected gambles. We develop a framework based on a small but powerful axiom schema and present two instantiations of this schema.

Accepting & Rejecting Gambles Consider an agent faced with uncertainty and risk, formalized using a linear space of gambles. We envisage an elicitation procedure where the agent is asked to state whether she would *accept* a gamble—and its possibly negative outcome—, *reject* it—if she considers it unreasonable to accept—, or remain uncommitted.

The agent's set of acceptable gambles together with the set of gambles she rejects form her *assessment*.

In terms of statements, a gamble can fall into one of four categories: only accepted, only rejected, *unresolved*—neither accepted nor rejected—, or *confusing*—both accepted and rejected.

Axiom: No Confusion We judge confusion to be a situation that has to be avoided.

Axiom *template*: Background Model Per problem domain, a set of acceptable gambles and a set of rejected gambles can be fixed; these form the *background model*, which has to be combined with the agent's own assessment.

Axiom *template*: Deductive Closure The problem domain's assumptions about the nature of the gamble payoffs—typically about the utility in which these are expressed—determines a *deductive extension* rule for accepted gambles. Starting from an assessment, this rule generates a *deductively closed* assessment.

Axiom: No Limbo Deductive Closure does have more of an impact than is apparent at first sight: An unresolved gamble may, were it accepted, cause confusion after applying deductive extension. We say that such unresolved gambles are *in limbo*, as they can only be rejected without leading to confusion. To obtain a *model*, the gambles in the limbo of a deductively closed assessment are therefore rejected.

Instantiations Consider the case where (i) the background model is defined by accepting uniformly nonnegative gambles and rejecting those that are uniformly negative, and (ii) gambles are assumed to be expressed in a linear utility, so that deductive extension coincides with taking the positive linear hull. Then our axiom schema generates a theory that generalizes Walley's (1991) theory of imprecise probability.

If instead we take the convex hull operation as the deductive extension—allowing some forms of nonlinear utility—our axiom schema generates a theory that generalizes Föllmer and Schied's (2002) theory of convex risk measures and Pelessoni and Vicig's (2005) imprecise probabilistic reformulation thereof.

References

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