The CONEstrip Algorithm

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Avoiding sure loss

- Finite possibility space Ω,
- Linear vector space $\mathcal{L} \coloneqq [\Omega \to \mathbb{R}]$,
- Finite set of gambles $\mathcal{K} \Subset \mathcal{L}$,
- Lower prevision $\underline{P} \in [\mathcal{K} \to \mathbb{R}]$,
- Set of marginal gambles $\mathcal{A} \coloneqq \{h - \underline{P}h : h \in \mathcal{K}\}$.

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$$\text{find} \quad \lambda \in \mathbb{R}^{\mathcal{A}},$$

$$\text{subject to} \quad \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g \lessdot 0 \quad \text{and} \quad \lambda \geq 0.$$

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$$\begin{aligned} \text{find} \quad & \lambda \in \mathbb{R}^{\mathcal{A}}, \\ \text{subject to} \quad & \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g \leq -1 \quad \text{and} \quad \lambda \geq 0. \end{aligned}$$

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- Indicator function 1_B of an event $B \subseteq \Omega$; $1_\omega \coloneqq 1_{\{\omega\}}$ for $\omega \in \Omega$.

$$\text{find} \quad (\lambda, \mu) \in \mathbb{R}^{\mathcal{A}} \times \mathbb{R}^{\Omega},$$
$$\text{subject to} \quad \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g + \sum_{\omega \in \Omega} \mu_\omega \cdot 1_\omega = 0 \quad \text{and} \quad \lambda \geq 0 \quad \text{and} \quad \mu \geq 1.$$

Natural extension

- Set of almost desirable gambles $\mathcal{A} \Subset \mathcal{L}$,
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&\text{maximize} && \alpha \in \mathbb{R}, \\
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subject to $\sum_{g \in \mathcal{A}} \lambda_g \cdot g + \sum_{\omega \in \Omega} \mu_\omega \cdot 1_\omega + \alpha = f$ and $\lambda \geq 0$ and $\mu \geq 0$.

Avoiding partial loss

- Set of (finite) events Ω^*,
- Finite set of (gamble, event)-pairs $\mathcal{N} \Subset \mathcal{L} \times \Omega^*$,
- Conditional lower prevision $\underline{P} \in [\mathcal{N} \to \mathbb{R}]$,
- Set of (conditional marginal gamble, event)-pairs

$$\mathcal{B} \coloneqq \Big\{ \big([h - \underline{P}(h|B)] \cdot 1_B, B \big) : (h, B) \in \mathcal{N} \Big\}.$$

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find $(\lambda, \varepsilon) \in \mathbb{R}^{\mathcal{B}} \times \mathbb{R}^{\mathcal{B}}$,

subject to $\sum_{(g,B)\in\mathcal{B}} \lambda_{g,B} \cdot [g + \varepsilon_{g,B} \cdot 1_B] \leq 0$ and $\lambda > 0$ and $\varepsilon \gtrdot 0$.

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$$\begin{aligned} \text{find} \quad & (\lambda, \nu, \mu) \in \mathbb{R}^{\mathcal{B}} \times (\mathbb{R}^{\mathcal{B}} \times \mathbb{R}^{\mathcal{B}}) \times \mathbb{R}^{\Omega}, \\ \text{subject to} \quad & \textstyle\sum_{(g,B) \in \mathcal{B}} \lambda_{g,B} \cdot [\nu_{g,B,g} \cdot g + \nu_{g,B,B} \cdot 1_B] + \sum_{\omega \in \Omega} \mu_\omega \cdot 1_\omega = 0 \\ \text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0 \quad \text{and} \quad \mu \geq 0. \end{aligned}$$

Conditional natural extension

- Set of (conditional almost desirable gamble, event)-pairs

$$\mathcal{B} \Subset \mathcal{L} \times \Omega^*,$$

- (Gamble, event)-pair of interest $(f, C) \in \mathcal{L} \times \Omega^*$.

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$$\begin{aligned} &\text{maximize} && \alpha \in \mathbb{R}, \\ &\text{subject to} && f \cdot 1_C - \alpha \cdot 1_C \geq \textstyle\sum_{(g,B)\in\mathcal{B}} \lambda_{g,B} \cdot [g + \varepsilon_{g,B} \cdot 1_B] \\ &\text{and to} && \lambda \geq 0 \quad \text{and} \quad \varepsilon \gg 0. \end{aligned}$$

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maximize $\alpha \in \mathbb{R}$,
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and to $\lambda \geq 0$ and $\varepsilon > 0$.

maximize $\alpha \in \mathbb{R}$,
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and to $\lambda \geq 0$ and $\nu > 0$ and $\mu \geq 0$.

Representation of general cones

Represent a finitary *general* cone as a *convex closure* of a finite number of finitary *open* cones.

$$\underline{\mathcal{R}} \coloneqq \left\{ \sum_{\mathcal{D}\in\mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} \nu_{\mathcal{D},g} \cdot g : \lambda > 0, \nu \gtrdot 0 \right\} \quad \text{for} \quad \mathcal{R} \Subset \mathcal{L}^*.$$

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Definition

An ajar cone $\mathcal{C}$ is finitary iff its closure $\mathrm{cl}\,\mathcal{C}$ is finitary and the intersection of $\mathcal{C}$ with each of $\mathrm{cl}\,\mathcal{C}$'s facets is a finitary (open, closed, or ajar) cone.

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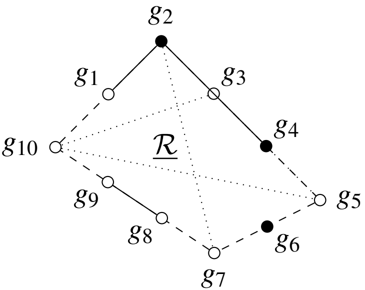
Theorem

$\underline{\mathcal{R}}$ is a finitary general cone for every $\mathcal{R} \Subset \mathcal{L}^{*}$.

Representation of general cones: illustration



$$\mathcal{R} \coloneqq \big\{ \{g_3, g_5, g_{10}\}, \{g_1, g_2\}, \{g_2, g_7\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}, \{g_6\} \big\}.$$

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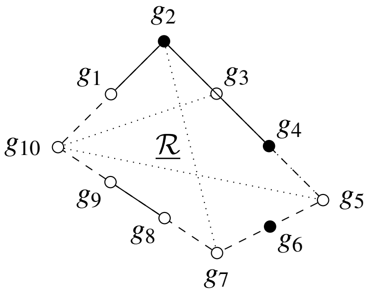

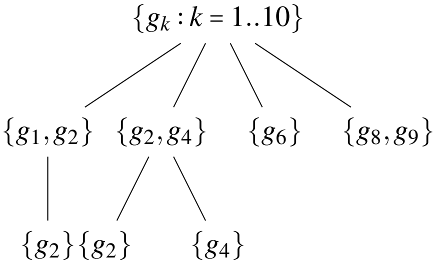

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Cone-in-facet representation:

$$\big\{\{g_k : k = 1..10\}, \{g_1, g_2\}, \{g_2, g_4\}, \{g_6\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}\big\}.$$

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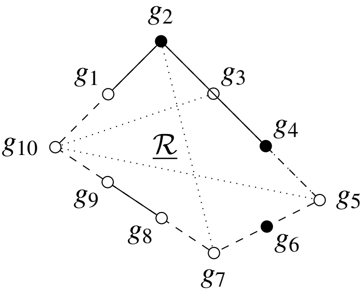

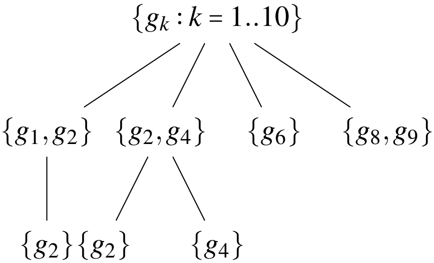

$$\mathcal{R} := \{\{g_3, g_5, g_{10}\}, \{g_1, g_2\}, \{g_2, g_7\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}, \{g_6\}\}.$$

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$$\{\{g_k : k = 1..10\}, \{g_1, g_2\}, \{g_2, g_4\}, \{g_6\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}\}.$$

Less redundant representation:

$$\{\{g_5, g_7, g_{10}\}, \{g_1, g_2\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}, \{g_6\}\}.$$

Formulation of the general problem

Given a general cone represented by $\mathcal{R} \Subset \mathcal{L}^*$ and a gamble $h \in \mathcal{L}$, we wish to

$$
\begin{aligned}
\text{find} \quad & (\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \times_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}} \\
\text{subject to} \quad & \textstyle\sum_{\mathcal{D} \in \mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g \in \mathcal{D}} \nu_{\mathcal{D},g} \cdot g = h \\
\text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0
\end{aligned}
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Given a general cone represented by $\mathcal{R} \Subset \mathcal{L}^*$ and a gamble $h \in \mathcal{L}$, we wish to

$$\begin{aligned} \text{find} \quad & (\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \times_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}} \\ \text{or maximize an affine function of} \quad & \mu \coloneqq (\lambda_{\mathcal{D}} \cdot \nu_{\mathcal{D},g} : \mathcal{D} \in \mathcal{R}, g \in \mathcal{D}), \\ \text{subject to} \quad & \textstyle\sum_{\mathcal{D} \in \mathcal{R}} \sum_{g \in \mathcal{D}} \mu_{\mathcal{D},g} \cdot g = h \\ \text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0 \end{aligned}$$

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find	$(\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \times_{\mathcal{D}\in\mathcal{R}} \mathbb{R}^{\mathcal{D}}$
or maximize an affine function of	$\mu \coloneqq (\lambda_{\mathcal{D}} \cdot \nu_{\mathcal{D},g} : \mathcal{D} \in \mathcal{R}, g \in \mathcal{D})$,
subject to	$\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \cdot g = h$
and to	$\lambda > 0$ and $\nu \gtrdot 0$
and to	possibly additional linear constraints on μ.

WLOG $h = 0$ in feasibility problem

find $(\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}}$

subject to $\sum_{\mathcal{D} \in \mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g \in \mathcal{D}} \nu_{\mathcal{D},g} \cdot g = h$

and to $\lambda > 0$ and $\nu \geqslant 0$

and to possibly … $\mu \coloneqq (\lambda_{\mathcal{D}} \cdot \nu_{\mathcal{D},g} : \mathcal{D} \in \mathcal{R}, g \in \mathcal{D})$.

WLOG $h = 0$ in feasibility problem

$$\begin{aligned}
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\text{and to} \quad & \text{possibly} \ldots \ \mu \coloneqq (\lambda_{\mathcal{D}} \cdot \nu_{\mathcal{D},g} : \mathcal{D} \in \mathcal{R}, g \in \mathcal{D}).
\end{aligned}$$

$$\begin{aligned}
\text{find} \quad & (\lambda, \nu) \in \mathbb{R}^{\mathcal{R}\cup\{\{-h\}\}} \times \textstyle\times_{\mathcal{D}\in\mathcal{R}\cup\{\{-h\}\}} \mathbb{R}^{\mathcal{D}}, \\
\text{subject to} \quad & \textstyle\sum_{\mathcal{D}\in\mathcal{R}\cup\{\{-h\}\}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} \nu_{\mathcal{D},g} \cdot g = 0 \\
\text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0 \quad \text{and} \quad \mu_{\{-h\},-h} = \lambda_{\{-h\}} \cdot \nu_{\{-h\},-h} \geq 1 \\
\text{and to} \quad & \text{possibly} \ldots \ \mu \coloneqq (\lambda_{\mathcal{D}} \cdot \nu_{\mathcal{D},g} : \mathcal{D} \in \mathcal{R}, g \in \mathcal{D}).
\end{aligned}$$

Blunt topological closure

find $(\lambda, v) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D}\in\mathcal{R}} \mathbb{R}^{\mathcal{D}}$

subject to $\sum_{\mathcal{D}\in\mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} v_{\mathcal{D},g} \cdot g = 0$

and to $\lambda > 0$ and $v \gtrdot 0$ and possibly. . .

Blunt topological closure

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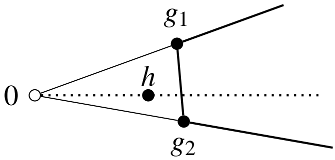
and to $\lambda > 0$ and $\nu \gtrdot 0$ and possibly...

find $\mu \in \bigtimes_{\mathcal{D}\in\mathcal{R}} \mathbb{R}^{\mathcal{D}}$,

subject to $\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \cdot g = 0$ and $\mu \geq 0$ and possibly...

and to $\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \geq 1$.



Topological interior

find	$(\lambda, v) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}}$
subject to	$\sum_{\mathcal{D} \in \mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g \in \mathcal{D}} v_{\mathcal{D},g} \cdot g = 0$
and to	$\lambda > 0$ and $v \gtrdot 0$ and possibly. . .

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find $(\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}}$
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and to $\lambda > 0$ and $\nu \gtrdot 0$ and possibly...

find $\mu \in \bigtimes_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}}$,
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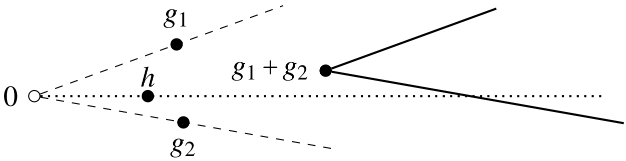


The CONEstrip algorithm

We can solve the general feasibility problem with arbitrary $\mathcal{R} \Subset \mathcal{L}^*$ and $h \coloneqq 0$ with the following algorithm:

1. maximize $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}}$,
   subject to $\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \cdot g = 0$ and $\mu \geq 0$ and possibly…
   and to $0 \leq \tau \leq 1$ and $\forall \mathcal{D} \in \mathcal{R} : \tau_{\mathcal{D}} \leq \mu_{\mathcal{D}}$ and $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}} \geq 1$.

2. a. If there is no feasible solution, then the problem is infeasible.
   b. Otherwise set $\mathcal{S} \coloneqq \{\mathcal{D} \in \mathcal{R} : \tau_{\mathcal{D}} > 0\}$; τ is equal to 1 on $\mathcal{S}$:
      i. If $\forall \mathcal{D} \in \mathcal{R} \setminus \mathcal{S} : \mu_{\mathcal{D}} = 0$, then the general problem is feasible.
      ii. Otherwise, return to step 1 with $\mathcal{R}$ replaced by $\mathcal{S}$.

The CONEstrip algorithm

We can solve the general feasibility problem with arbitrary $\mathcal{R} \Subset \mathcal{L}^*$ and $h := 0$ with the following algorithm:

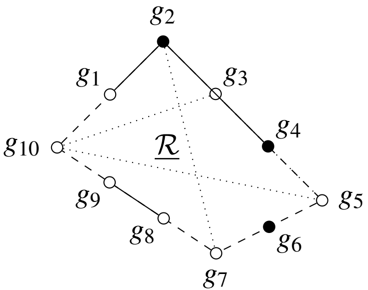
1. maximize $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}}$,
   subject to $\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \cdot g = 0$ and $\mu \geq 0$ and possibly...
   and to $0 \leq \tau \leq 1$ and $\forall \mathcal{D} \in \mathcal{R} : \tau_{\mathcal{D}} \leq \mu_{\mathcal{D}}$ and $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}} \geq 1$.

2. a. If there is no feasible solution, then the problem is infeasible.
   b. Otherwise set $\mathcal{S} := \{\mathcal{D} \in \mathcal{R} : \tau_{\mathcal{D}} > 0\}$; τ is equal to 1 on $\mathcal{S}$:
      i. If $\forall \mathcal{D} \in \mathcal{R} \setminus \mathcal{S} : \mu_{\mathcal{D}} = 0$, then the general problem is feasible.
      ii. Otherwise, return to step 1 with $\mathcal{R}$ replaced by $\mathcal{S}$.

Proposition

The claims made in the CONEstrip algorithm are veracious and it terminates after at most $|\mathcal{R}| - 1$ iterations.

The CONEstrip algorithm: illustration



$$\mathcal{R} \coloneqq \{\{g_3, g_5, g_{10}\}, \{g_1, g_2\}, \{g_2, g_7\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}, \{g_6\}\}$$

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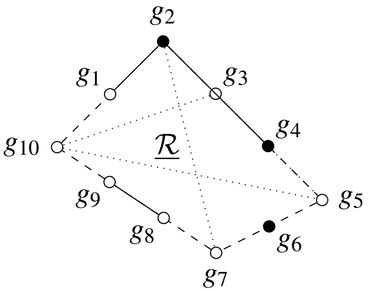

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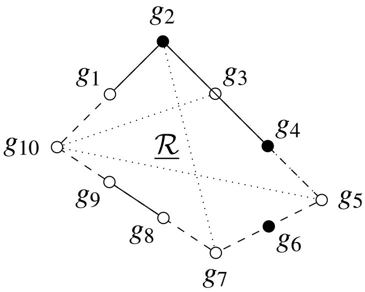
We show that $g_3 \in \underline{\mathcal{R}}$:

(It. 1) $\mathcal{S} = \mathcal{R}$, $\tau_{\{g_2\}} = \tau_{\{g_4\}} = \tau_{\{-g_3\}} = 1$, and possibly $\mu_{\{g_3, g_5, g_{10}\}} > 0$

(It. 2) $\mathcal{S} = \big\{\{g_2\}, \{g_4\}, \{-g_3\}\big\}$ and $\tau = 1$.

The CONEstrip algorithm: illustration



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We show that $g_1 \notin \underline{\mathcal{R}}$:

(It. 1) $\mathcal{S} = \mathcal{R}$, $\tau_{\{g_2\}} = \tau_{\{g_1, g_2\}} = \tau_{\{-g_1\}} = 1$, and necessarily $\mu_{\{g_3, g_5, g_{10}\}, g_{10}} > 0$,

(It. 2) $\mathcal{S} = \big\{\{g_2\}, \{g_1, g_2\}, \{-g_1\}\big\}$, infeasible.

Optimization problems

We can solve the general optimization problem with arbitrary $\mathcal{R} \Subset \mathcal{L}^*$ and $h \in \mathcal{L}$ with the following algorithm:

1. Apply the CONEstrip algorithm to $\mathcal{R} \cup \{-h\}$ with $\mu_{\{-h\},-h} \geq 1$ as an additional constraint; if feasible, continue to the next step with the terminal set $\mathcal{S}$.
2.

$$\begin{aligned} \text{maximize an affine function of} \quad & \mu, \\ \text{subject to} \quad & \textstyle\sum_{\mathcal{D} \in \mathcal{S}} \sum_{g \in \mathcal{D}} \mu_{\mathcal{D},g} \cdot g = h \\ \text{and to} \quad & \mu \geq 0 \quad \text{and} \quad \text{possibly}\ldots \end{aligned}$$

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