The CONEstrip Algorithm

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Avoiding sure loss

- Finite possibility space Ω,
- Linear vector space $\mathcal{L} \coloneqq [\Omega \to \mathbb{R}]$,
- Finite set of gambles $\mathcal{K} \Subset \mathcal{L}$,
- Lower prevision $\underline{P} \in [\mathcal{K} \to \mathbb{R}]$,
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$$\text{find} \quad \lambda \in \mathbb{R}^{\mathcal{A}},$$
$$\text{subject to} \quad \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g \lessdot 0 \quad \text{and} \quad \lambda \geq 0.$$

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$$\begin{aligned} \text{find} \quad & \lambda \in \mathbb{R}^{\mathcal{A}}, \\ \text{subject to} \quad & \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g \lessdot 0 \quad \text{and} \quad \lambda \geq 0. \end{aligned}$$

- Indicator function 1_B of an event $B \subseteq \Omega$; $1_\omega \coloneqq 1_{\{\omega\}}$ for $\omega \in \Omega$.

$$\begin{aligned} \text{find} \quad & (\lambda, \mu) \in \mathbb{R}^{\mathcal{A}} \times \mathbb{R}^{\Omega}, \\ \text{subject to} \quad & \textstyle\sum_{g \in \mathcal{A}} \lambda_g \cdot g + \sum_{\omega \in \Omega} \mu_\omega \cdot 1_\omega = 0 \quad \text{and} \quad \lambda \geq 0 \quad \text{and} \quad \mu \geq 1. \end{aligned}$$

Avoiding partial loss

- Set of (finite) events Ω^*,
- Finite set of (gamble, event)-pairs $\mathcal{N} \Subset \mathcal{L} \times \Omega^*$,
- Conditional lower prevision $\underline{P} \in [\mathcal{N} \to \mathbb{R}]$,
- Set of (conditional marginal gamble, event)-pairs

$$\mathcal{B} := \Big\{ \big([h - \underline{P}(h|B)] \cdot 1_B, B \big) : (h, B) \in \mathcal{N} \Big\}.$$

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find $(\lambda, \varepsilon) \in \mathbb{R}^{\mathcal{B}} \times \mathbb{R}^{\mathcal{B}}$,

subject to $\sum_{(g,B)\in\mathcal{B}} \lambda_{g,B} \cdot [g + \varepsilon_{g,B} \cdot 1_B] \leq 0$ and $\lambda > 0$ and $\varepsilon \gtrdot 0$.

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$$\mathcal{B} \coloneqq \Big\{\big([h - \underline{P}(h|B)] \cdot 1_B, B\big) : (h, B) \in \mathcal{N}\Big\}.$$

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$$\begin{aligned} \text{find} \quad & (\lambda, \nu, \mu) \in \mathbb{R}^{\mathcal{B}} \times (\mathbb{R}^{\mathcal{B}} \times \mathbb{R}^{\mathcal{B}}) \times \mathbb{R}^{\Omega}, \\ \text{subject to} \quad & \textstyle\sum_{(g,B)\in\mathcal{B}} \lambda_{g,B} \cdot [\nu_{g,B,g} \cdot g + \nu_{g,B,B} \cdot 1_B] + \sum_{\omega\in\Omega} \mu_\omega \cdot 1_\omega = 0 \\ \text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0 \quad \text{and} \quad \mu \geq 0. \end{aligned}$$

Representation of finitary general cones

As a *convex closure* of a finite number of finitary *open* cones:

$$\underline{\mathcal{R}} \coloneqq \left\{ \sum_{\mathcal{D}\in\mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} \nu_{\mathcal{D},g} \cdot g : \lambda > 0, \nu \gtrdot 0 \right\} \quad \text{for} \quad \mathcal{R} \Subset \mathcal{L}^*.$$

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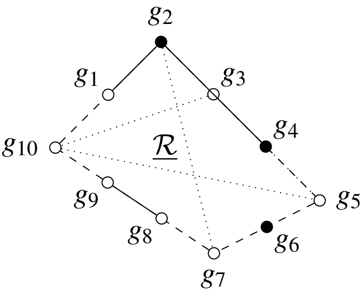

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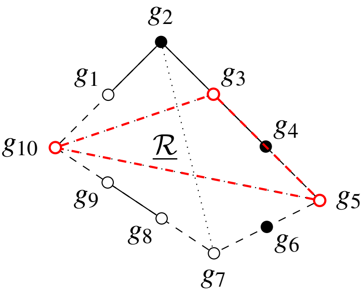

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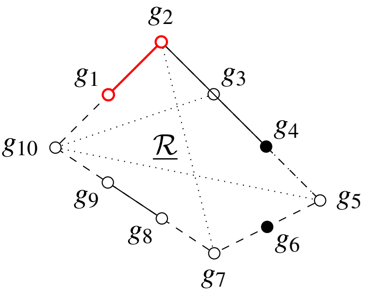

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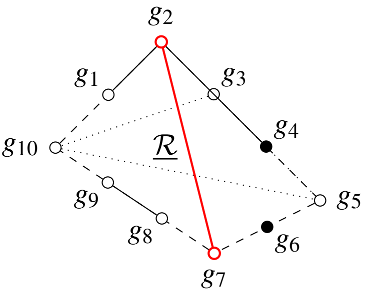

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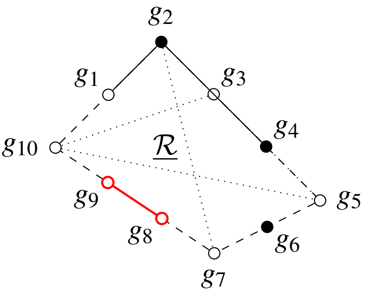

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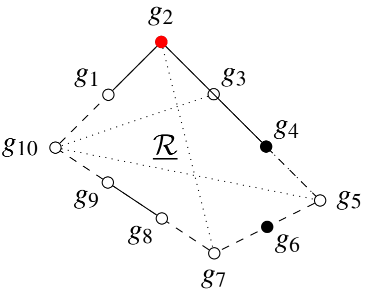

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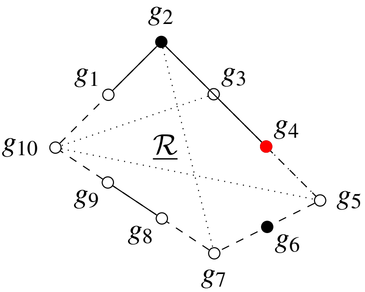

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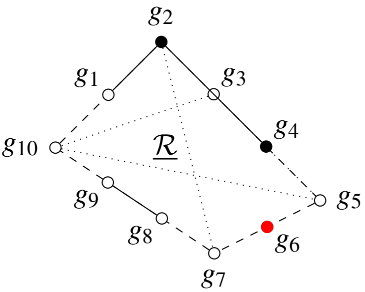

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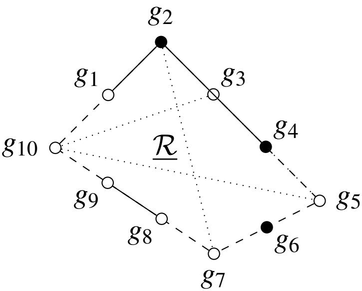

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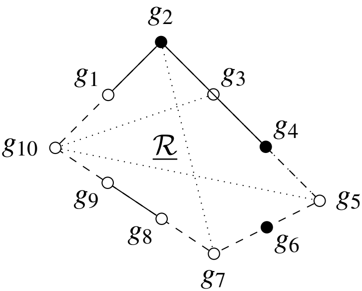

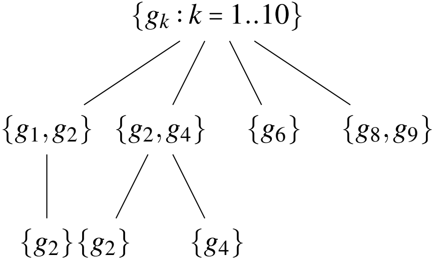

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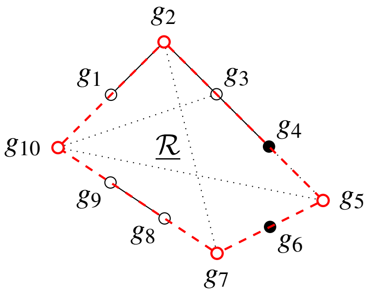

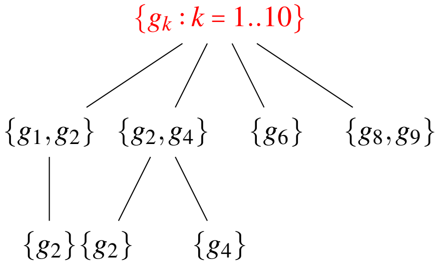

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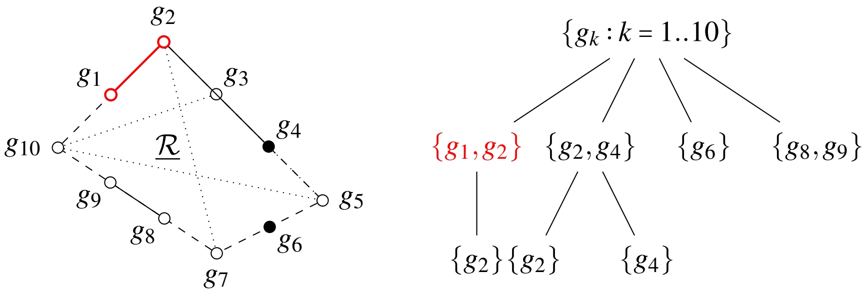

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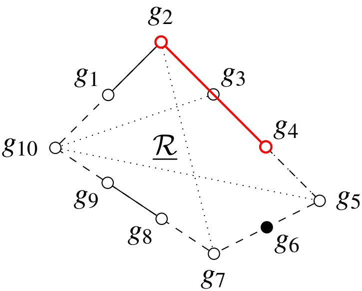

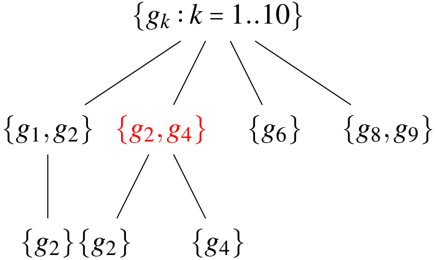

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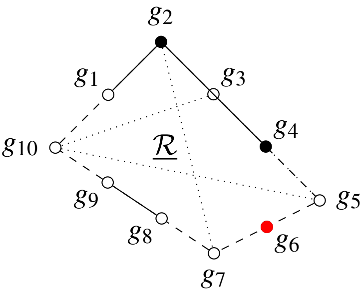

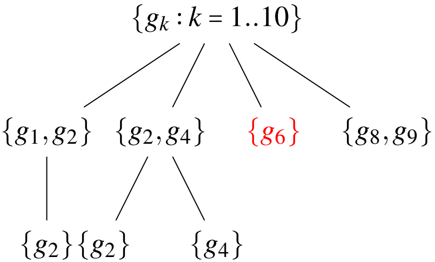

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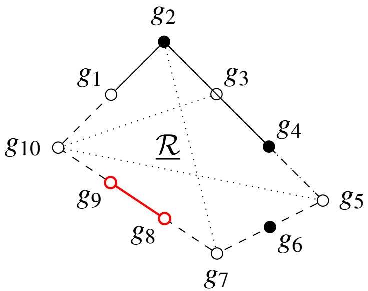

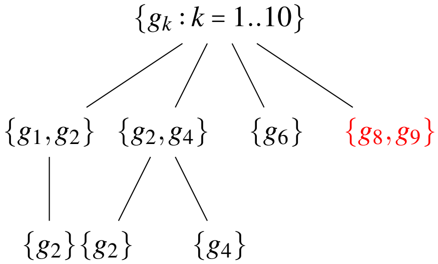

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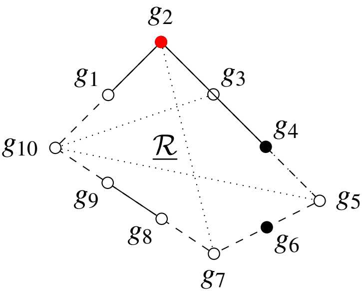

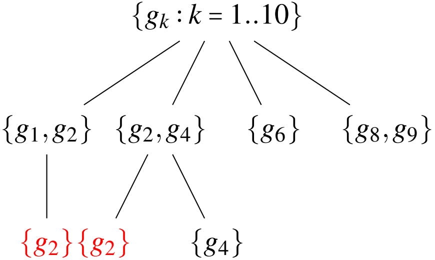

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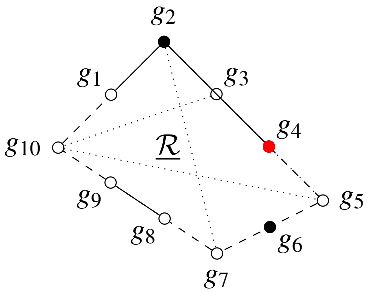

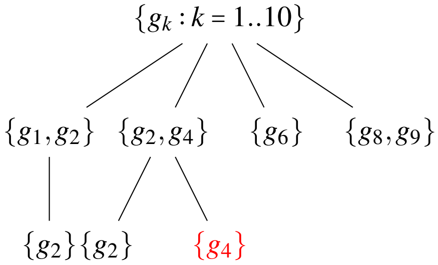

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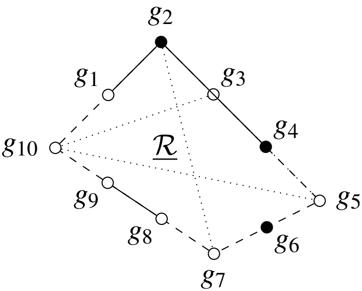
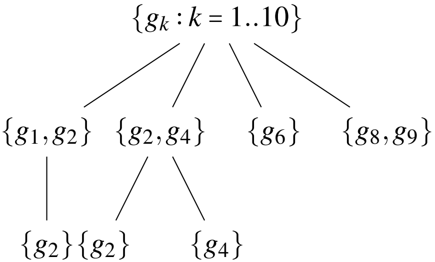
Representation of finitary general cones

As a *convex closure* of a finite number of finitary *open* cones:

$$\underline{\mathcal{R}} \coloneqq \left\{ \sum_{\mathcal{D}\in\mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} \nu_{\mathcal{D},g} \cdot g : \lambda > 0, \nu \gtrdot 0 \right\} \quad \text{for} \quad \mathcal{R} \Subset \mathcal{L}^*.$$





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Formulation of the general problem

Given a general cone represented by $\mathcal{R} \Subset \mathcal{L}^*$ and a gamble $h \in \mathcal{L}$, we wish to

$$\begin{aligned} \text{find} \quad & (\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}} \\ \text{subject to} \quad & \textstyle\sum_{\mathcal{D} \in \mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g \in \mathcal{D}} \nu_{\mathcal{D},g} \cdot g = h \\ \text{and to} \quad & \lambda > 0 \quad \text{and} \quad \nu \gtrdot 0 \end{aligned}$$

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WLOG $h = 0$.

Approximating the problem: Blunt topological closure

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&\text{and to} && \textstyle\sum_{\mathcal{D} \in \mathcal{R}} \sum_{g \in \mathcal{D}} \mu_{\mathcal{D},g} \geq 1.
\end{aligned}
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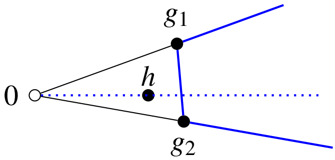

Approximating the problem: Topological interior

find	$(\lambda, \nu) \in \mathbb{R}^{\mathcal{R}} \times \bigtimes_{\mathcal{D}\in\mathcal{R}} \mathbb{R}^{\mathcal{D}}$
subject to	$\sum_{\mathcal{D}\in\mathcal{R}} \lambda_{\mathcal{D}} \cdot \sum_{g\in\mathcal{D}} \nu_{\mathcal{D},g} \cdot g = 0$
and to	$\lambda > 0$ and $\nu \geqslant 0$

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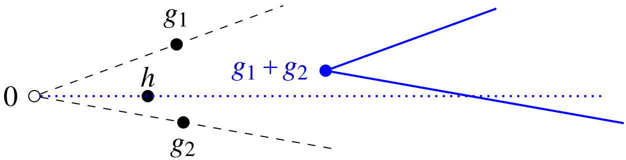
find $\quad \mu \in \times_{\mathcal{D} \in \mathcal{R}} \mathbb{R}^{\mathcal{D}},$

subject to $\quad \sum_{\mathcal{D} \in \mathcal{R}} \sum_{g \in \mathcal{D}} \mu_{\mathcal{D},g} \cdot g = 0 \quad$ and $\quad \mu \geq 1$



The CONEstrip algorithm

We can solve the general problem with arbitrary $\mathcal{R} \Subset \mathcal{L}^*$ with the following algorithm:

1. maximize $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}}$,

   subject to $\sum_{\mathcal{D}\in\mathcal{R}} \sum_{g\in\mathcal{D}} \mu_{\mathcal{D},g} \cdot g = 0$ and $\mu \geq 0$

   and to $0 \leq \tau \leq 1$ and $\forall \mathcal{D} \in \mathcal{R} : \tau_{\mathcal{D}} \leq \mu_{\mathcal{D}}$ and $\sum_{\mathcal{D}\in\mathcal{R}} \tau_{\mathcal{D}} \geq 1$.

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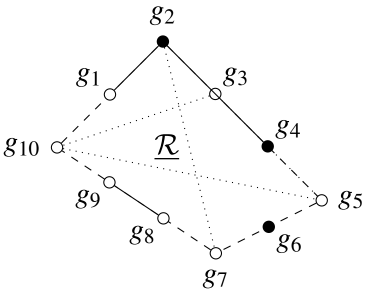
      ii. Otherwise, return to step 1 with $\mathcal{R}$ replaced by $\mathcal{S}$.

The CONEstrip algorithm: illustration



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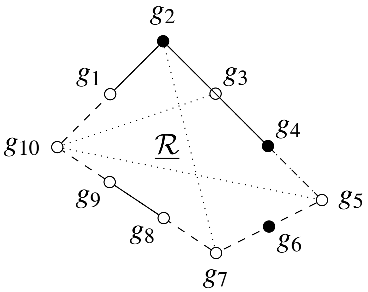


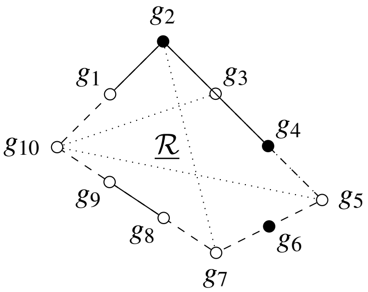
$$\mathcal{R} := \{\{g_3, g_5, g_{10}\}, \{g_1, g_2\}, \{g_2, g_7\}, \{g_8, g_9\}, \{g_2\}, \{g_4\}, \{g_6\}\}$$

We show that $g_3 \in \underline{\mathcal{R}}$:

(It. 1) $\mathcal{S} = \mathcal{R}$, $\tau_{\{g_2\}} = \tau_{\{g_4\}} = \tau_{\{-g_3\}} = 1$, and possibly $\mu_{\{g_3,g_5,g_{10}\},g_3} > 0$

(It. 2) $\mathcal{S} = \{\{g_2\}, \{g_4\}, \{-g_3\}\}$ and $\tau = 1$.

The CONEstrip algorithm: illustration



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We show that $g_1 \notin \underline{\mathcal{R}}$:

(It. 1) $\mathcal{S} = \mathcal{R}$, $\tau_{\{g_2\}} = \tau_{\{g_1,g_2\}} = \tau_{\{-g_1\}} = 1$, and necessarily $\mu_{\{g_3,g_5,g_{10}\},g_{10}} > 0$,

(It. 2) $\mathcal{S} = \big\{\{g_2\}, \{g_1, g_2\}, \{-g_1\}\big\}$, infeasible.

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- Integrating CONEstrip with a specific linear programming solver might allow for a practical increase in efficiency.