Sets of desirable gambles and their connection to probabilistically-flavored models for uncertainty

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Context & assumptions

Possibility space $\mathcal{X}$ outcomes experiment

We—an intentional system uncertain about outcome experiment

Goal model our uncertainty/beliefs/information & use this model for reasoning

Gambles payoff depends on outcome, bounded real-valued function on $\mathcal{X}$, set of gambles $\mathcal{L}(\mathcal{X})$

Utility linear and precise
Gambles

\[ f(b) = \frac{3}{5} \]

\[ f = \left( \frac{1}{2}, \frac{3}{5} \right) \]

\[ f(a) = \frac{1}{2} \]
Gambles

\[ f(a) = \frac{1}{2}, \quad f(b) = \frac{3}{5} \]

\[ I_a = (1, 0), \quad I_b = (0, 1), \quad I_{\{a, b\}} = (1, 1) \]
Gambles

\[ f = \left( -\frac{2}{3}, \frac{5}{6}, \frac{5}{6} \right) \]
Desirable gambles

Gamble $f$ desirable when we accept the transaction

(i) the experiment’s outcome $x$ is determined
(ii) our capital is changed by $f(x)$

Our uncertainty model set of desirable gambles
Outline

Reasoning about and with sets of desirable gambles
- Rationality criteria
- Assessments avoiding partial (or sure) loss
- Coherent sets of desirable gambles
- Natural extension

Relationships with other, nonequivalent models

Derived coherent sets of desirable gambles

Combining sets of desirable gambles

Partial preference orders
Constructive rationality criteria
It is reasonable to require that a set of desirable gambles \( \mathcal{D} \subseteq \mathcal{L}(\mathcal{X}) \) satisfies

Positive scaling: \( \lambda > 0 \Rightarrow \lambda \mathcal{D} = \mathcal{D} \),

Addition: \( \mathcal{D} + \mathcal{D} = \mathcal{D} \).
Constructive rationality criteria

It is reasonable to require that a set of desirable gambles $\mathcal{D} \subseteq \mathcal{L}(\mathcal{X})$ satisfies

Positive scaling: $\lambda > 0 \Rightarrow \lambda \mathcal{D} = \mathcal{D}$,

Addition: $\mathcal{D} + \mathcal{D} = \mathcal{D}$.

They extend an *assessment* $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ to

$$\text{posi}(\mathcal{A}) := \left\{ \sum_{k=1}^{n} \lambda_k f_k : \lambda_k > 0 \land f_k \in \mathcal{L}(\mathcal{X}) \land n \in \mathbb{N} \right\}$$
Constraining rationality criteria

Comparing gambles the ordinary vector ordering is defined by

\[ f \geq g \iff (f - g) \in \mathcal{L}_0^+(\mathcal{X}) \iff \inf(f - g) \geq 0 \]
\[ f > g \iff (f - g) \in \mathcal{L}^+(\mathcal{X}) \iff \inf(f - g) \geq 0 \land \sup(f - g) > 0 \]
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It is reasonable to require that a set of desirable gambles \( \mathcal{D} \subseteq \mathcal{L}(\mathcal{X}) \) satisfies

- Accepting partial gain: \( \mathcal{L}^+(\mathcal{X}) \subseteq \mathcal{D} \)
- Avoiding partial loss: \( \mathcal{D} \cap \mathcal{L}^-(\mathcal{X}) = \emptyset \)
Constraining rationality criteria

Comparing gambles the ordinary vector ordering is defined by

\[ f \geq g \iff (f - g) \in \mathcal{L}_0^+(\mathcal{X}) \iff \inf(f - g) \geq 0 \]
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If \( \mathcal{D} \subseteq \mathcal{L}(\mathcal{X}) \) accepts partial gain and avoids partial loss, then it also satisfies

Accepting sure gain: \( \text{int}(\mathcal{L}^+(\mathcal{X})) \subseteq \mathcal{D} \)
Avoiding sure loss: \( \mathcal{D} \cap \text{int}(\mathcal{L}^-(\mathcal{X})) = \emptyset \)
Assessments & partial loss

An assessment $A \subseteq \mathcal{L}(\mathcal{X})$ avoids partial loss iff

$$\text{posi}(A) \cap \mathcal{L}^-(\mathcal{X}) = \emptyset$$
Assessments & partial loss

An assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ avoids partial loss iff

$$\text{posi}(\mathcal{A}) \cap \mathcal{L}^-(\mathcal{X}) = \emptyset$$

An assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ incurs partial loss iff

$$\text{posi}(\mathcal{A}) \cap \mathcal{L}^-(\mathcal{X}) \neq \emptyset$$
Coherent sets of desirable gambles

**Coherence** A set of desirable gambles \( \mathcal{D} \subseteq \mathcal{L}(\mathcal{X}) \) is coherent if it satisfies all four rationality criteria.

**Geometry** It is a convex cone containing the positive orthant \( \mathcal{L}^+(\mathcal{X}) \), but excluding the negative orthant \( \mathcal{L}^-(\mathcal{X}) \).

![Diagram of a convex cone](image)

Set of coherent sets \( \mathbb{D}(\mathcal{X}) \)
Coherent extensions

Coherent extensions of an assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ Any encompassing coherent set of desirable gambles

Set of coherent extensions $\mathbb{D}_\mathcal{A} := \{\mathcal{D} \in \mathbb{D}(\mathcal{X}) : \mathcal{A} \subseteq \mathcal{D}\}$
Coherent extensions

Coherent extensions of an assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ Any encompassing coherent set of desirable gambles

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Inclusion based partial order of extensions that are more/less committal
Coherent extensions

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Set of coherent extensions $\mathbb{D}_\mathcal{A} := \{\mathcal{D} \in \mathbb{D}(\mathcal{X}) : \mathcal{A} \subseteq \mathcal{D}\}$

Inclusion based partial order of extensions that are more/less committal
Natural extension

Given the constructive rationality criteria and accepting partial gains, there is a *natural extension* of an assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$:

\[
\mathcal{E}(\mathcal{A}) := \text{posi}(\mathcal{A} \cup \mathcal{L}^+(\mathcal{X}))
\]
Natural extension
Given the constructive rationality criteria and accepting partial gains, there is a natural extension of an assessment $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$:

$$\mathcal{E}(\mathcal{A}) := \text{posi}(\mathcal{A} \cup \mathcal{L}^+(\mathcal{X}))$$

Natural Extension Theorem
The natural extension $\mathcal{E}(\mathcal{A})$ of $\mathcal{A} \subseteq \mathcal{L}(\mathcal{X})$ coincides with its least committal coherent extension $\bigcap \mathcal{D}_\mathcal{A}$ if and only if $\mathcal{A}$ avoids partial loss.

Natural extension is the prime tool for deductive inference in desirability.
Outline

Reasoning about and with sets of desirable gambles

Relationships with other, nonequivalent models
- Linear previsions
- Credal sets
- To lower & upper previsions
- From lower previsions

Derived coherent sets of desirable gambles

Combining sets of desirable gambles

Partial preference orders
Linear previsions

Linear previsions . . .

- are positive linear normed expectation operators
- provide fair prices for gambles in $\mathcal{L}(\mathcal{X})$
- are equivalent to (finitely additive) probability measures and, on finite $\mathcal{X}$, to probability mass functions
Linear previsions

Linear previsions . . .

- are positive linear normed expectation operators
- provide fair prices for gambles in \( L(\mathcal{X}) \)
- are equivalent to (finitely additive) probability measures and, on finite \( \mathcal{X} \), to probability mass functions
- belong to the closed convex set \( \mathbb{P}(\mathcal{X}) \) which is, for finite \( \mathcal{X} \), the unit simplex spanned by the degenerate previsions (or \( \{0, 1\} \)-valued probability mass functions)
Linear previsions

- are positive linear normed expectation operators
- provide fair prices for gambles in $\mathcal{L}(\mathcal{X})$
- are equivalent to (finitely additive) probability measures and, on finite $\mathcal{X}$, to probability mass functions
- belong to the closed convex set $\mathbb{P}(\mathcal{X})$ which is, for finite $\mathcal{X}$, the unit simplex spanned by the degenerate previsions (or $\{0,1\}$-valued probability mass functions)
- provide probabilities for events, as fair prices for their indicators
From linear previsions to sets of desirable gambles

Given a linear prevision $P \in \mathbb{P}(\mathcal{X})$, gambles with a strictly positive fair price are strictly desirable:

$$\mathcal{D}_P := \mathcal{E}(\mathcal{A}_P), \quad \text{with} \quad \mathcal{A}_P := \{f \in \mathcal{L}(\mathcal{X}) : P(f) > 0\}$$
From linear previsions to sets of desirable gambles

Given a linear prevision $P \in \mathbb{P}(\mathcal{X})$, gambles with a strictly positive fair price are strictly desirable:

$$\mathcal{D}_P := \mathcal{E}(\mathcal{A}_P), \quad \text{with} \quad \mathcal{A}_P := \{ f \in \mathcal{L}(\mathcal{X}) : P(f) > 0 \}$$

Observations:
- $\{ f \in \mathcal{L}(\mathcal{X}) : P(f) = 0 \}$ is a linear subspace of $\mathcal{L}(\mathcal{X})$
- So $\mathcal{A}_P$ is an open halfspace
- Except in a few borderline cases, so is $\mathcal{D}_P$
From credal sets to sets of desirable gambles

A credal set is a set of linear previsions

Given a credal set $\mathcal{M} \subseteq \mathbb{P}(\mathcal{X})$, gambles with a strictly positive fair price for every linear prevision in the credal set are strictly desirable:

$$D_{\mathcal{M}} := \mathcal{E}(A_{\mathcal{M}}), \quad \text{with} \quad A_{\mathcal{M}} := \{f \in \mathcal{L}(\mathcal{X}) : (\forall P \in \mathcal{M} : P(f) > 0)\}$$

$$= \bigcap_{P \in \mathcal{M}} A_P$$
From credal sets to sets of desirable gambles

A credal set is a set of linear previsions

Given a credal set $\mathcal{M} \subseteq \mathbb{P}(\mathcal{X})$, gambles with a strictly positive fair price for every linear prevision in the credal set are strictly desirable:

$$D_M := \mathcal{E}(A_M), \quad \text{with} \quad A_M := \{ f \in \mathcal{L}(\mathcal{X}) : (\forall P \in \mathcal{M} : P(f) > 0) \}$$

$$= \bigcap_{P \in \mathcal{M}} A_P$$

Observations:

- Each prevision gives rise to a linear constraint in gamble space
- Constraints from linear previsions strictly in the convex hull of $\mathcal{M}$ are redundant
- So the border structure of $\mathcal{M}$ is uniquely important
From credal sets to sets of desirable gambles: example
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\[ \frac{2}{3} f(a) + \frac{1}{3} f(b) > 0 \]
From credal sets to sets of desirable gambles: example
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From credal sets to sets of desirable gambles: example

\[ f(a) + f(b) > 0 \]

\[
\begin{align*}
P_a &= (\frac{1}{3}, 0, \frac{2}{3}) \\
P_b &= (\frac{1}{3}, 2, \frac{1}{3}) \\
P_c &= (\frac{1}{6}, 1, \frac{2}{3}) \\
(\frac{1}{6}, 1, \frac{1}{6}) & \in \mathcal{M} \\
(\frac{2}{3}, \frac{1}{3}, 0) & \in \mathcal{M} \\
(\frac{1}{6}, \frac{5}{12}, \frac{5}{12}) & \notin \mathcal{M} \\
(\frac{1}{6}, \frac{2}{3}, \frac{1}{6}) & \notin \mathcal{M}
\end{align*}
\]
From credal sets to sets of desirable gambles: example

\[ f(a) + f(b) > 0 \]

\[ P_a P_b P_c \]

\[ \mathcal{M} \]

\[ (\frac{1}{3}, 0, \frac{2}{3}) \]
\[ (\frac{1}{6}, \frac{1}{3}, \frac{2}{3}) \]
\[ (\frac{1}{6}, \frac{1}{2}, \frac{1}{3}) \]
\[ (\frac{1}{6}, \frac{2}{3}, \frac{1}{6}) \]

\[ (\frac{2}{3}, 0, \frac{1}{3}) \]

\((\frac{1}{6}, \frac{5}{12}, \frac{5}{12})\)
\[(\frac{1}{6}, \frac{1}{6}, \frac{2}{3})\]
\[(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})\]
From credal sets to sets of desirable gambles: example

![Diagram showing the transition from credal sets to sets of desirable gambles with points labeled as follows:

- $P_a$ with coordinates $(\frac{2}{3}, \frac{1}{3}, 0)$
- $P_b$ with coordinates $(\frac{1}{3}, \frac{2}{3}, 0)$
- $P_c$ with coordinates $(\frac{1}{3}, 0, \frac{2}{3})$

The diagram also shows points $I_a$ and $I_b$ with coordinates $(\frac{1}{6}, \frac{5}{12}, \frac{5}{12})$ and $(\frac{1}{6}, \frac{1}{6}, \frac{2}{3})$, respectively, and a shaded region labeled $\mathcal{M}$ with vertices at $(\frac{1}{6}, \frac{1}{2}, \frac{1}{3})$, $(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$, and $(\frac{1}{3}, 0, \frac{2}{3})$. The diagram illustrates the correspondence between these sets.]
From credal sets to sets of desirable gambles: example

\[
(f(a) + f(b)) > 0
\]

\[
P_a \cup P_b \cup P_c
\]
From credal sets to sets of desirable gambles: example
From credal sets to sets of desirable gambles: example
From credal sets to sets of desirable gambles: example
From desirable gambles to credal sets

Given a coherent set of strictly desirable gambles $\mathcal{D} \subset \mathcal{L}(\mathcal{X})$, we derive the associated credal set as follows:

$$\mathcal{M}_D := \bigcap_{f \in \mathcal{D}} \{ P \in \mathbb{P}(\mathcal{X}) : P(f) \geq 0 \}.$$  

Credal Set Proposition

The credal set $\mathcal{M}_D \subseteq \mathbb{P}(\mathcal{X})$ associated to a coherent set of desirable gambles $\mathcal{D} \subset \mathcal{L}(\mathcal{X})$ is closed and convex.
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \]

\[ 2I_a - \frac{1}{3} \]

\[ I_{\{a,b\}} - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \]
From desirable gambles to credal sets: example

\[
I_{\{a, c\}} - \frac{1}{3} \geq 2I_{a} - \frac{1}{3} \geq I_{\{b, c\}} - \frac{1}{3} \geq I_{\{a, b\}} - \frac{1}{3} \geq I_{a} - \frac{1}{3} \geq 0 \geq I_{a} - \frac{1}{3} \geq 0 \geq I_{a} - \frac{1}{3} \geq 0
\]
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \quad 2I_a - \frac{1}{3} \quad I_{\{a,b\}} - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \quad I_b \quad D \]

\[ P(\{a\}) \geq \frac{1}{6} \]
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \]

\[ I_{\{a,b\}} - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \]

\[ 2I_a - \frac{1}{3} \]

\[ P(\{a, b\}) \geq \frac{1}{3} \]

\[ P(\{a\}) \geq \frac{1}{6} \]
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \]

\[ 2I_a - \frac{1}{3} \]

\[ I_{\{a,b\}} - \frac{1}{3} \]

\[ I_c \]

\[ D \]

\[ \hat{P}({\{b\}}) \geq 0 \]

\[ P({\{a, b\}}) \geq \frac{1}{3} \]

\[ P({\{a\}}) \geq \frac{1}{6} \]
From desirable gambles to credal sets: example

\[ P(\{a\}) \geq \frac{1}{6} \]
\[ P(\{b\}) \geq 0 \]
\[ P(\{a, b\}) \geq \frac{1}{3} \]
\[ P(\{b, c\}) \geq \frac{1}{3} \]
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \]

\[ 2I_a - \frac{1}{3} \]

\[ I_{\{a,b\}} - \frac{1}{3} \]

\[ D \]

\[ P(\{b\}) \geq 0 \]

\[ P(\{a, b\}) \geq \frac{1}{3} \]

\[ P(\{c\}) \geq 0 \]

\[ P(\{a\}) \geq \frac{1}{6} \]

\[ P(\{b, c\}) \geq \frac{1}{3} \]

\[ P(\{a, c\}) \geq \frac{1}{6} \]
From desirable gambles to credal sets: example

\[
I_{\{a,c\}} - \frac{1}{3} \leq I_{\{b,c\}} - \frac{1}{3} \\
2I_a - \frac{1}{3} \leq I_{\{a,b\}} - \frac{1}{3} \\
I_a \leq \frac{1}{3} \leq I_b \leq \frac{1}{3} \leq I_c \\
\]

\[
P(\{b\}) \geq 0 \leq P(\{a,b\}) \geq \frac{1}{3} \leq P(\{a\}) \geq \frac{1}{6} \leq P(\{a,c\}) \geq \frac{1}{3} \leq P(\{b,c\}) \geq \frac{1}{3}
\]
From desirable gambles to credal sets: example

\[ I_{\{a, c\}} - \frac{1}{3} \leq I_{\{b, c\}} - \frac{1}{3} \leq I_{\{a, b\}} - \frac{1}{3} \leq I_a - \frac{1}{3} \leq I_{\{a, c\}} - \frac{1}{3} \leq 0 \leq I_{\{a\}} \leq I_{\{b, c\}} - \frac{1}{3} \]

\[ P(\{b\}) \geq 0 \]

\[ P(\{a, b\}) \geq \frac{1}{3} \]

\[ P(\{c\}) \geq 0 \]

\[ P(\{a\}) \geq \frac{1}{6} \]

\[ P(\{b, c\}) \geq \frac{1}{3} \]

\[ P(\{a, c\}) \geq \frac{1}{3} \]
From desirable gambles to credal sets: example

\[ I_{\{a,c\}} - \frac{1}{3} \]

\[ 2I_a - \frac{1}{3} \]

\[ I_{\{b,c\}} - \frac{1}{3} \]

\[ P(\{c\}) \geq 0 \]

\[ P(\{a, b\}) \geq \frac{1}{3} \]

\[ P(\{a\}) \geq \frac{1}{6} \]

\[ P(\{b, c\}) \geq \frac{1}{3} \]

\[ P(\{a, c\}) \geq \frac{1}{3} \]
Lower & upper previsions

Lower previsions . . .

▶ are positive superlinear normed expectation operators
▶ provide supremum acceptable buying prices for gambles in $\mathcal{L}(\mathcal{X})$
▶ provide lower probabilities for events

Upper previsions . . .

▶ are positive sublinear normed expectation operators
▶ provide infimum acceptable selling prices for gambles in $\mathcal{L}(\mathcal{X})$
▶ provide upper probabilities for events
Lower & upper previsions

Lower previsions . . .
- are positive superlinear normed expectation operators
- provide supremum acceptable buying prices for gambles in $\mathcal{L}(\mathcal{X})$
- provide lower probabilities for events

Upper previsions . . .
- are positive sublinear normed expectation operators
- provide infimum acceptable selling prices for gambles in $\mathcal{L}(\mathcal{X})$
- provide upper probabilities for events

Prices can be seen as constant gambles, which are trivially linearly ordered
From sets of desirable gambles to lower & upper previsions

Given a coherent set of strictly desirable gambles $\mathcal{D} \subset \mathcal{L}(\mathcal{X})$, we use comparisons with constant gambles to derive lower and upper previsions:

$$
\begin{align*}
P_D(f) &= \sup \{ \alpha \in \mathbb{R} : f \succ \alpha \} = \sup \{ \alpha \in \mathbb{R} : f - \alpha \in \mathcal{D} \} \\
P_D(f) &= \inf \{ \beta \in \mathbb{R} : \beta \succ f \} = \inf \{ \beta \in \mathbb{R} : \beta - f \in \mathcal{D} \}
\end{align*}
$$

Conjugacy:

$$P_D(f) = -P_D(-f)$$
and

$$P_D(A) = 1 - P_D(A^c)$$
From sets of desirable gambles to lower & upper previsions

Given a coherent set of strictly desirable gambles $D \subset \mathcal{L}(\mathcal{X})$, we use comparisons with constant gambles to derive lower and upper previsions:

$$P_D(f) := \sup \{ \alpha \in \mathbb{R} : f \succ \alpha \} = \sup \{ \alpha \in \mathbb{R} : f - \alpha \in D \}$$
From sets of desirable gambles to lower & upper previsions

Given a coherent set of strictly desirable gambles $\mathcal{D} \subset \mathcal{L}(\mathcal{X})$, we use comparisons with constant gambles to derive lower and upper previsions:

$$P_D(f) := \sup\{\alpha \in \mathbb{R}: f \succ \alpha\} = \sup\{\alpha \in \mathbb{R}: f - \alpha \in \mathcal{D}\}$$

$$\bar{P}_D(f) := \inf\{\beta \in \mathbb{R}: \beta \succ f\} = \inf\{\beta \in \mathbb{R}: \beta - f \in \mathcal{D}\}$$
From sets of desirable gambles to lower & upper previsions

Given a coherent set of strictly desirable gambles $\mathcal{D} \subset \mathcal{L}(\mathcal{X})$, we use comparisons with constant gambles to derive lower and upper previsions:

$$P_\mathcal{D}(f) := \sup\{\alpha \in \mathbb{R} : f \succ \alpha\} = \sup\{\alpha \in \mathbb{R} : f - \alpha \in \mathcal{D}\}$$

$$\overline{P}_\mathcal{D}(f) := \inf\{\beta \in \mathbb{R} : \beta \succ f\} = \inf\{\beta \in \mathbb{R} : \beta - f \in \mathcal{D}\}$$

Conjugacy: $\overline{P}_\mathcal{D}(f) = -P_\mathcal{D}(-f)$ and $\overline{P}_\mathcal{D}(A) = 1 - P_\mathcal{D}(A^c)$
From lower previsions to sets of desirable gambles
Given a lower prevision $P$ defined on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$, how do we derive an associated set of desirable gambles?
From lower previsions to sets of desirable gambles

Given a lower prevision $P$ defined on $\mathcal{K} \subseteq \mathcal{L} (\mathcal{X})$, how do we derive an associated set of desirable gambles?

**Constant additivity** is a rationality requirement derived from coherent sets of marginal gambles: $P_D(f + \alpha) = P_D(f) + \alpha$

A marginal gamble is a gamble with lower prevision zero derived from any gamble in $\mathcal{K}$ by constant additivity: $G_P(f) := f - P(f)$
From lower previsions to sets of desirable gambles

Given a lower prevision $P$ defined on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$, how do we derive an associated set of desirable gambles?

**Constant additivity** is a rationality requirement derived from coherent sets of marginal gambles: $P_D(f + \alpha) = P_D(f) + \alpha$

A marginal gamble is a gamble with lower prevision zero derived from any gamble in $\mathcal{K}$ by constant additivity: $G_P(f) := f - P(f)$

Use marginal as marginally desirable gambles, i.e., border gambles:

$$\mathcal{D}_P := \mathcal{E}(\mathcal{A}_P) \quad \text{with} \quad \mathcal{A}_P := \mathcal{G}_P + \mathbb{R}^+ \quad \text{and} \quad \mathcal{G}_P := \mathcal{G}_P(\mathcal{K})$$
From lower previsions to sets of desirable gambles
Given a lower prevision $P$ defined on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$, how do we derive an associated set of desirable gambles?

Constant additivity is a rationality requirement derived from coherent sets of marginal gambles: $P_D(f + \alpha) = P_D(f) + \alpha$

A marginal gamble is a gamble with lower prevision zero derived from any gamble in $\mathcal{K}$ by constant additivity: $G_P(f) := f - P(f)$

Use marginal as marginally desirable gambles, i.e., border gambles:

$$\mathcal{D}_P := \mathcal{E}(A_P) \quad \text{with} \quad A_P := G_P + \mathbb{R}^+ \quad \text{and} \quad G_P := G_P(\mathcal{K})$$
From lower previsions to sets of desirable gambles

Given a lower prevision $\mathbb{P}$ defined on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$, how do we derive an associated set of desirable gambles?

**Constant additivity** is a rationality requirement derived from coherent sets of marginal gambles: $P_D(f + \alpha) = P_D(f) + \alpha$

A marginal gamble is a gamble with lower prevision zero derived from any gamble in $\mathcal{K}$ by constant additivity: $G_{\mathbb{P}}(f) := f - \mathbb{P}(f)$

Use marginal as marginally desirable gambles, i.e., border gambles:

$$D_{\mathbb{P}} := \mathcal{E}(A_{\mathbb{P}}) \quad \text{with} \quad A_{\mathbb{P}} := G_{\mathbb{P}} + \mathbb{R}^+ \quad \text{and} \quad G_{\mathbb{P}} := G_{\mathbb{P}}(\mathcal{K})$$
Translating desirability concepts to lower previsions

Avoiding sure loss for a lower prevision \( P \) on \( \mathcal{K} \subseteq \mathcal{L}(\mathcal{X}) \)
corresponds to \( A_P \) avoiding sure (or partial) loss:

\[
\forall g \in \text{posi}(\mathcal{G}_P) : \sup g \geq 0.
\]
Translating desirability concepts to lower previsions

Avoiding sure loss for a lower prevision $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$
corresponds to $A_P$ avoiding sure (or partial) loss:

$$\forall g \in \text{posi}(G_P) : \sup g \geq 0.$$  

Natural extension $E$ of a lower previsions $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ to $\mathcal{L}(\mathcal{X})$
corresponds to $D_P$:

$$E(f) = \sup \{ \alpha \in \mathbb{R} : (\exists g \in \text{posi}(G_P) : f - \alpha \geq g) \}$$
Translating desirability concepts to lower previsions

Avoiding sure loss for a lower prevision $\mathcal{P}$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$

corresponds to $\mathcal{A}_\mathcal{P}$ avoiding sure (or partial) loss:

$$\forall g \in \text{posi}(\mathcal{G}_\mathcal{P}) : \sup g \geq 0.$$ 

Natural extension $\mathcal{E}$ of a lower previsions $\mathcal{P}$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ to $\mathcal{L}(\mathcal{X})$

corresponds to $\mathcal{D}_\mathcal{P}$:

$$\mathcal{E}(f) = \sup\{\alpha \in \mathbb{R} : (\exists g \in \text{posi}(\mathcal{G}_\mathcal{P}) : f - \alpha \geq g)\}$$
Translating desirability concepts to lower previsions

Avoiding sure loss for a lower prevision $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$
corresponds to $A_P$ avoiding sure (or partial) loss:

$$\forall g \in \text{posi}(G_P) : \sup g \geq 0.$$

Natural extension $E$ of a lower previsions $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ to $\mathcal{L}(\mathcal{X})$
corresponds to $D_P$:

$$E(f) = \sup\{\alpha \in \mathbb{R} : (\exists g \in \text{posi}(G_P) : f - \alpha \geq g)\}$$
Translating desirability concepts to lower previsions (c’d)
Natural extension $E$ of a lower previsions $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ to $\mathcal{L}(\mathcal{X})$ corresponds to $D_P$:

$$E(f) = \sup \{ \alpha \in \mathbb{R} : (\exists g \in \text{posi}(G_P) : f - \alpha \geq g) \}$$
Translating desirability concepts to lower previsions (c’d)

Natural extension $E$ of a lower previsions $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ to $\mathcal{L}(\mathcal{X})$ corresponds to $\mathcal{D}_P$:

$$E(f) = \sup\{\alpha \in \mathbb{R} : (\exists g \in \text{posi}(\mathcal{G}_P) : f - \alpha \geq g)\}$$

Coherence for lower previsions $P$ on $\mathcal{K} \subseteq \mathcal{L}(\mathcal{X})$ corresponds to coherence of $\mathcal{D}_P$:

$$\forall f \in \mathcal{G}_P : \forall g \in \text{posi}(\mathcal{G}_P) : \sup(g - f) \geq 0$$
Outline

Reasoning about and with sets of desirable gambles

Relationships with other, nonequivalent models

Derived coherent sets of desirable gambles
- Gamble space transformations
- Conditional sets of desirable gambles
- Conditional lower previsions
- Marginal sets of desirable gambles

Combining sets of desirable gambles

Partial preference orders
Transformation of a set of desirable gambles

\[ \mathcal{D}_\Gamma := \{ h \in \mathcal{L}(\mathcal{Z}) : \Gamma h \in \mathcal{D} \} \]
Transformation of a set of desirable gambles

\[ \mathcal{D}_\Gamma := \{ h \in \mathcal{L}(\mathcal{Z}) : \Gamma h \in \mathcal{D} \} \]

- \( \Gamma : \mathcal{L}(\{d, b\}) \to \mathcal{L}(\{a, b\}) \)
- \( (\Gamma h)(a) = \frac{1}{2} h(d) \) and \( (\Gamma h)(b) = h(b) \)
Taking a slice of a set of desirable gambles
Taking a slice of a set of desirable gambles

$\Gamma_1 : \mathcal{L} \{c, d\} \to \mathcal{L} \{a, b, c\}$

$\Gamma_1 h(a) = \Gamma_1 h(b) = h(d)$ and $\Gamma_1 h(c) = h(c)$
Taking a slice of a set of desirable gambles

\[ I_c \left( \left( -\frac{1}{3}, 0, \frac{4}{3} \right) \right) \]

\[ 2I_b - \frac{1}{3} \left( \left( \frac{1}{6}, \frac{5}{4}, -\frac{5}{12} \right) \right) \]

\[ \Gamma_1 : \mathcal{L}(\{c, d\}) \rightarrow \mathcal{L}(\{a, b, c\}) \]

\[ (\Gamma_1 h)(a) = (\Gamma_1 h)(b) = h(d) \text{ and } (\Gamma_1 h)(c) = h(c) \]

\[ \Gamma_2 : \mathcal{L}(\{c, d\}) \rightarrow \mathcal{L}(\{a, b, c\}) \]

\[ (\Gamma_2 h)(a) = \frac{3}{4} h(d), \ (\Gamma_2 h)(b) = \frac{1}{4} h(d) \text{ and } (\Gamma_2 h)(c) = h(c) \]
Conditional sets of desirable gambles

Conditioning event $B \subseteq \mathcal{X}$ is what the experiment’s outcome is assumed to belong to.

Contingent gambles are those for which, if $B$ does not occur, status quo is maintained.

Transformation $\mathcal{T}_{B^c}$ maps gambles on $B$ to contingent gambles on $\mathcal{X}$:

$$(\mathcal{T}_{B^c} h)(x) = \begin{cases} h(x), & x \in B, \\ 0, & x \in B^c, \end{cases}$$
Conditional sets of desirable gambles

Conditioning event $B \subseteq \mathcal{X}$ is what the experiment’s outcome is assumed to belong to.

Contingent gambles are those for which, if $B$ does not occur, status quo is maintained.

Transformation $\upharpoonright_{B^c}$ maps gambles on $B$ to contingent gambles on $\mathcal{X}$:

$$(\upharpoonright_{B^c} h)(x) = \begin{cases} h(x), & x \in B, \\ 0, & x \in B^c, \end{cases}$$

Conditional set of desirable gambles Given a set of desirable gambles $\mathcal{D} \subseteq \mathcal{L}(\mathcal{X})$, the set of desirable gambles conditional on $B$ is

$$\mathcal{D}|B := \mathcal{D}_{\upharpoonright_{B^c}} = \{ h \in \mathcal{L}(B) : \upharpoonright_{B^c} h \in \mathcal{D} \}$$
Conditional sets of desirable gambles: example
Conditional sets of desirable gambles: example

\[ D \{ a, b \} \]

\[ D \{ a, b \} \]

\[ (\frac{4}{3}, 0, -\frac{1}{3}) \]

\[ (\frac{5}{3}, 0, -\frac{2}{3}) \]

\[ (1, \frac{5}{9}, -\frac{5}{9}) \]

\[ (\frac{1}{6}, \frac{5}{4}, -\frac{5}{12}) \]

\[ (\frac{1}{3}, 0, \frac{4}{3}) \]

\[ 2I_b - \frac{1}{3} \]

\[ I_a \]

\[ I_b \]

\[ I_c \]
Conditional sets of desirable gambles: example

\[ D \mid \{a, b\} \]

\[ D \mid \{b, c\} \]
Conditional sets of desirable gambles: example

\[ I_a, I_b, I_c \]

\[ D \]

\[ 2I_b - \frac{1}{3} \]

\[ (\frac{4}{3}, 0, -\frac{1}{3}) \]

\[ (\frac{5}{3}, 0, -\frac{2}{3}) \]

\[ (1, \frac{5}{9}, -\frac{5}{9}) \]

\[ (\frac{1}{6}, \frac{5}{4}, -\frac{5}{12}) \]

\[ (-\frac{1}{3}, 0, \frac{4}{3}) \]

\[ D\{a, b\} \]

\[ D\{b, c\} \]

\[ D\{c, a\} \]

\[ (\frac{25}{18}, -\frac{7}{18}) \]

\[ (\frac{4}{3}, -\frac{1}{3}) \]
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?

\[ Pf := \min \left\{ \frac{3}{4} f(a) + \frac{1}{4} f(b), \frac{1}{3} f(a) + \frac{2}{3} f(b), f(c) \right\} \]
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?

\[ Pf := \min \left\{ \frac{3}{4} f(a) + \frac{1}{4} f(b), \frac{1}{3} f(a) + \frac{2}{3} f(b), f(c) \right\} \]
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?

\[ P_f := \min\{\frac{3}{4}f(a) + \frac{1}{4}f(b), \frac{1}{3}f(a) + \frac{2}{3}f(b), f(c)\} \]

\[ P(\cdot\{a, b\}) := P_{\mathcal{D}_P}\{a, b\} = \min \]
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?

- $Pf := \min\{\frac{3}{4}f(a) + \frac{1}{4}f(b), \frac{1}{3}f(a) + \frac{2}{3}f(b), f(c)\}$
- $P(\cdot|\{a, b\}) := P_{\mathcal{D}_P}\{a, b\} = \min$
- $\mathcal{R}_P := \mathcal{D}_P \cup \{f \in \text{cl}(\mathcal{D}_P) : \overline{P}(f) > 0\}$
Natural versus regular extension
When is the border structure of sets of strictly desirable gambles important?

\[
\begin{align*}
Pf & := \min\{\frac{3}{4}f(a) + \frac{1}{4}f(b), \frac{1}{3}f(a) + \frac{2}{3}f(b), f(c)\} \\
\overline{P}(\cdot|\{a, b\}) & := \overline{P}_{\overline{\mathcal{D}}_P}\{|a, b\} = \min \\
\overline{\mathcal{R}}_P & := \overline{\mathcal{D}}_P \cup \{f \in \text{cl}(\overline{\mathcal{D}}_P) : \overline{P}(f) > 0\} \\
\overline{R}(\cdot|\{a, b\}) & := \overline{P}_{\overline{\mathcal{R}}_P}\{|a, b\} = \min\{\frac{3}{4}f(a) + \frac{1}{4}f(b), \frac{1}{3}f(a) + \frac{2}{3}f(b)\}
\end{align*}
\]
Marginal sets of desirable gambles

Cartesian product possibility space $\mathcal{X} = \mathcal{Y} \times \mathcal{Z}$, focus on $\mathcal{Y}$-component (ignore $\mathcal{Z}$-component)

Cylindrical extension $\uparrow_{\mathcal{Z}}$ maps gambles from the source gamble space to its cartesian product with $\mathcal{L}(\mathcal{Z})$:

$$(\uparrow_{\mathcal{Z}} h)(y, z) = h(y)$$
Marginal sets of desirable gambles

Cartesian product possibility space $\mathcal{X} = \mathcal{Y} \times \mathcal{Z}$, focus on $\mathcal{Y}$-component (ignore $\mathcal{Z}$-component)

Cylindrical extension $\uparrow_{\mathcal{Z}}$ maps gambles from the source gamble space to its cartesian product with $\mathcal{L}(\mathcal{Z})$:

$$(\uparrow_{\mathcal{Z}} h)(y, z) = h(y)$$

Marginal set of desirable gambles Given a set of desirable gambles $\mathcal{D} \subseteq \mathcal{L}(\mathcal{Y} \times \mathcal{Z})$, its $\mathcal{Y}$-marginal is

$$\mathcal{D} \downarrow \mathcal{Y} := \mathcal{D}_{\uparrow_{\mathcal{Z}}} = \{ h \in \mathcal{L}(\mathcal{Y}) : \uparrow_{\mathcal{Z}} h \in \mathcal{D} \}$$
Outline

Reasoning about and with sets of desirable gambles

Relationships with other, nonequivalent models

Derived coherent sets of desirable gambles

Combining sets of desirable gambles
  ■ Joining compatible individuals
  ■ Marginal extension

Partial preference orders
Combining sets of desirable gambles: example

\[ (-\frac{1}{3}, 0, \frac{4}{3}) \]

\[ (\frac{4}{3}, 0, -\frac{1}{3}) \]

\[ (\frac{5}{3}, 0, -\frac{2}{3}) \]

\[ (1, \frac{5}{9}, -\frac{5}{9}) \]

\[ (\frac{1}{6}, \frac{5}{4}, -\frac{5}{12}) \]
Combining sets of desirable gambles: example

\[ A := \Gamma_1(\mathcal{D}_{\Gamma_1}) \cup \Gamma_2(\mathcal{D}_{\Gamma_2}) \cup \uparrow \{c\}(\mathcal{D}|\{a, b\}) \]
\[ \cup \uparrow \{a\}(\mathcal{D}|\{b, c\}) \]
\[ \cup \uparrow \{b\}(\mathcal{D}|\{c, a\}) \]
Combining sets of desirable gambles: example

\[ A := \Gamma_1(\mathcal{D}_{\Gamma_1}) \cup \Gamma_2(\mathcal{D}_{\Gamma_2}) \cup \mathcal{I}_{\{c\}}(\mathcal{D}|\{a, b\}) \]
\[ \cup \mathcal{I}_{\{a\}}(\mathcal{D}|\{b, c\}) \]
\[ \cup \mathcal{I}_{\{b\}}(\mathcal{D}|\{c, a\}) \]
Marginal extension

Separately specified conditional sets of desirable gambles have disjunct possibility spaces.

Separately coherent conditional sets of desirable gambles are separately specified and individually coherent.
Marginal extension

Separately specified conditional sets of desirable gambles have disjunct possibility spaces.

Separately coherent conditional sets of desirable gambles are separately specified and individually coherent.

Marginal Extension Theorem

Given a partition $\mathcal{B}$ of $\mathcal{X}$, a coherent $\mathcal{B}$-marginal $\mathcal{D}_B \subset \mathcal{L}(B)$, and separately coherent conditional sets of desirable gambles $\mathcal{D}|B \subset \mathcal{L}(B)$, $B \in \mathcal{B}$, then their combination $\mathcal{D} := \mathcal{E}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{X})$, with $\mathcal{A} := \Gamma_B(\mathcal{D}_B) \cup \bigcup_{B \in \mathcal{B}} \nmid_{B^c}(\mathcal{D}|B)$, is coherent as well.
Marginal extension

Separately specified conditional sets of desirable gambles have disjunct possibility spaces

Separately coherent conditional sets of desirable gambles are separately specified and individually coherent

Marginal Extension Theorem

Given a partition $\mathcal{B}$ of $\mathcal{X}$, a coherent $\mathcal{B}$-marginal $\mathcal{D}_B \subset \mathcal{L}(\mathcal{B})$, and separately coherent conditional sets of desirable gambles $\mathcal{D}|_B \subset \mathcal{L}(\mathcal{B})$, $B \in \mathcal{B}$, then their combination $\mathcal{D} := \mathcal{E}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{X})$, with $\mathcal{A} := \Gamma_B(\mathcal{D}_B) \cup \bigcup_{B \in \mathcal{B}} \mathcal{T}_B^c(\mathcal{D}|_B)$, is coherent as well.
Outline

Reasoning about and with sets of desirable gambles

Relationships with other, nonequivalent models

Derived coherent sets of desirable gambles

Combining sets of desirable gambles

Partial preference orders

- Strict preference
- Nonstrict preference
- Examples
Partial strict preference order

Strict preference $f \succ g$ if we are eager to exchange $g$ for $f$

Partial order The order does not have to be complete, $f \not\succ g \land g \not\succ f$ is possible

Strict desirability is strict preference over status quo, the zero gamble $0$:

$$f \succ g \iff f - g \succ 0 \iff f - g \in \mathcal{D}$$
Partial strict preference order

Strict preference $f \succ g$ if we are eager to exchange $g$ for $f$

Partial order The order does not have to be complete, $f \not\succ g \land g \not\succ f$ is possible

Strict desirability is strict preference over status quo, the zero gamble $0$:

$$f \succ g \iff f - g \succ 0 \iff f - g \in \mathcal{D}$$

Rationality criteria for strict preference relations $\succ$ on $\mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X})$:

- **Irreflexivity:** $f \not\succ f$
- **Transitivity:** $f \succ g \land g \succ h \Rightarrow f \succ h$
- **Mix-indep.:** $0 < \mu \leq 1 \Rightarrow (f \succ g \iff \mu f + (1 - \mu)h \succ \mu g + (1 - \mu)h)$
- **Monotonicity:** $f > g \Rightarrow f > g$
Partial strict preference order

Strict preference \( f \succ g \) if we are eager to exchange \( g \) for \( f \)

Partial order The order does not have to be complete, \( f \not\succ g \land g \not\succ f \) is possible

Strict desirability is strict preference over status quo, the zero gamble 0:

\[
f \succ g \iff f - g \succ 0 \iff f - g \in \mathcal{D}
\]

Rationality criteria for strict preference relations \( \succ \) on \( \mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X}) \):

Irreflexivity: \( f \not\succ f \)

Transitivity: \( f \succ g \land g \succ h \Rightarrow f \succ h \)

Mix-indep.: \( 0 < \mu \leq 1 \Rightarrow (f \succ g \iff \mu f + (1 - \mu) h \succ \mu g + (1 - \mu) h) \)

Monotonicity: \( f > g \Rightarrow f \succ g \)

Strengthening coherence criteria for sets of desirable gambles \( \mathcal{D} \):

Avoiding nonpositivity: \( 0 \notin \mathcal{D} \)
Partial nonstrict preference order

Nonstrict preference $f \succeq g$ if we are willing, i.e., not adverse, to exchange $g$ for $f$

Partial order The order does not have to be complete

Nonstrict desirability is nonstrict preference over status quo:

$$f \succeq g \iff f - g \succeq 0 \iff f - g \in \mathcal{D}$$
Partial nonstrict preference order

Nonstrict preference $f \succeq g$ if we are willing, i.e., not adverse, to exchange $g$ for $f$

Partial order The order does not have to be complete

Nonstrict desirability is nonstrict preference over status quo:

$$f \succeq g \iff f - g \succeq 0 \iff f - g \in \mathcal{D}$$

Rationality criteria for nonstrict preference relations $\succeq$ on $\mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X})$:

- Reflexivity: $f \succeq f$
- Transitivity: $g \succeq h \land f \succeq g \Rightarrow f \succeq h$
- Mix-indep.: $0 < \mu \leq 1 \Rightarrow (f \succeq g \iff \mu f + (1 - \mu)h \succeq \mu g + (1 - \mu)h)$
- Monotonicity: $f > g \Rightarrow f \succeq g \land g \not\succeq f$
Partial nonstrict preference order

Nonstrict preference $f \succeq g$ if we are willing, i.e., not adverse, to exchange $g$ for $f$

Partial order The order does not have to be complete

Nonstrict desirability is nonstrict preference over status quo:

$$f \succeq g \iff f - g \succeq 0 \iff f - g \in \mathcal{D}$$

Rationality criteria for nonstrict preference relations $\succeq$ on $\mathcal{L}(\mathcal{X}) \times \mathcal{L}(\mathcal{X})$:

- Reflexivity: $f \succeq f$
- Transitivity: $g \succeq h \land f \succeq g \Rightarrow f \succeq h$
- Mix-indep.: $0 < \mu \leq 1 \Rightarrow (f \succeq g \Leftrightarrow \mu f + (1 - \mu)h \succeq \mu g + (1 - \mu)h)$
- Monotonicity: $f > g \Rightarrow f \succeq g \land g \not\succeq f$

Strengthening coherence criteria for sets of desirable gambles $\mathcal{D}$:

Accepting nonnegativity: $\mathcal{L}^+_0(\mathcal{X}) \subseteq \mathcal{D}$
Strict vs. nonstrict

- Strict preference is more useful for decision making
Strict vs. nonstrict

- Strict preference is more useful for decision making.
- Advantages of nonstrict preference:
  
  **Indifference** is the equivalence relation defined by symmetric nonstrict preference:

  \[ f \equiv g \iff f \succ g \land g \succ f \]

  **Incomparability** is the irreflexive relation defined by symmetric nonstrict nonpreference:

  \[ f \bowtie g \iff f \not\succ g \land g \not\succ f \]
Strict vs. nonstrict

- Strict preference is more useful for decision making
- Advantages of nonstrict preference:
  - **Indifference** is the equivalence relation defined by symmetric nonstrict preference:
    \[ f \equiv g \iff f \succ g \land g \succ f \]
  - **Incomparability** is the irreflexive relation defined by symmetric nonstrict nonpreference:
    \[ f \nless g \iff f \not\succ g \land g \not\succ f \]

Example:

\[
\begin{array}{c}
\mathcal{K}_1 \\
\mathcal{K}_2 & \mathcal{K}_3 \\
0
\end{array}
\]

- \( \equiv \)-equivalence classes \( \mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3 \)
- intransitivity of \( \nless \):
  - \( \mathcal{K}_1 \nless \mathcal{K}_3 \) and \( \mathcal{K}_3 \nless \mathcal{K}_2 \), but \( \mathcal{K}_1 \nless \mathcal{K}_2 \)
Strict and nonstrict preferences: examples
Strict and nonstrict preferences: examples
Strict and nonstrict preferences: examples

\[ f \succ g \]

\[ f \preceq g \]

\[ f - g \]
Strict and nonstrict preferences: examples

$g - f \succ f - g$

$f \succsim g$

$g - f \succeq f - g$

$f \succ g$

$f - g \equiv g - f$

$f \succsim g$
Strict and nonstrict preferences: examples
Strict and nonstrict preferences: examples

\[ g - f \succ f \sim g \succ f - g \]

\[ g - f \succ f \sim g - f \succ f - g \]

\[ f \succ g \succ f - g \]

\[ f \succ g \succ f - g \]

\[ f \equiv g \]

\[ f \equiv g \]

\[ f \equiv g \]

\[ g - f \succ f \sim g - f \succ f - g \]

\[ g - f \succ f \sim g - f \succ f - g \]

\[ f \equiv g \]

\[ f \equiv g \]
Strict and nonstrict preferences: examples
Strict and nonstrict preferences: examples

\[ f \succ g \]
\[ f \succsim g \]
\[ f \preceq g \]

\[ f = g \]
\[ f \succeq g \]

\[ f \nsucc g \]
\[ f \nsuccsim g \]

\[ f \nsucceq g \]
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Full section outline

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