FE design optimization under uncertainty

as a

Constrained optimization problem under uncertainty

Gert de Cooman, Etienne Kerre,
Erik Quaeghebeur & Keivan Shariatmadar

FUM & SYSTeMS
Research Groups

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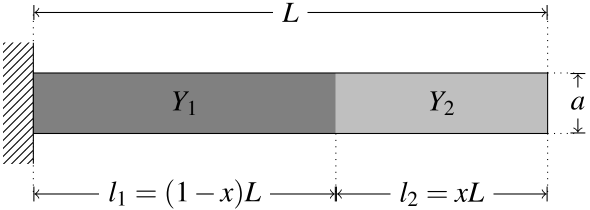
Toy problem: two-component massless rod



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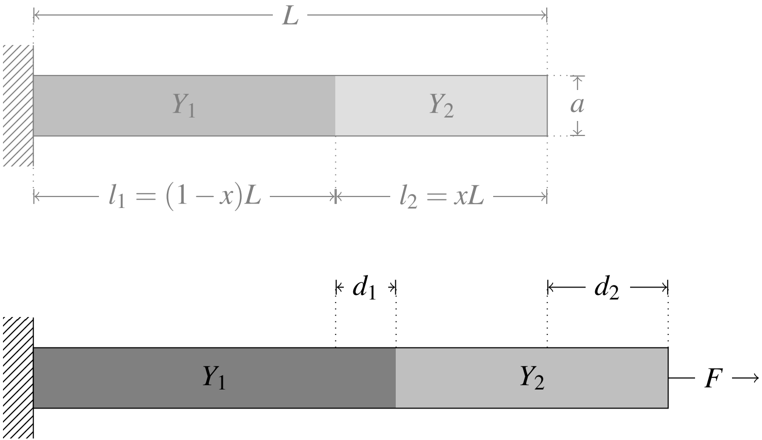

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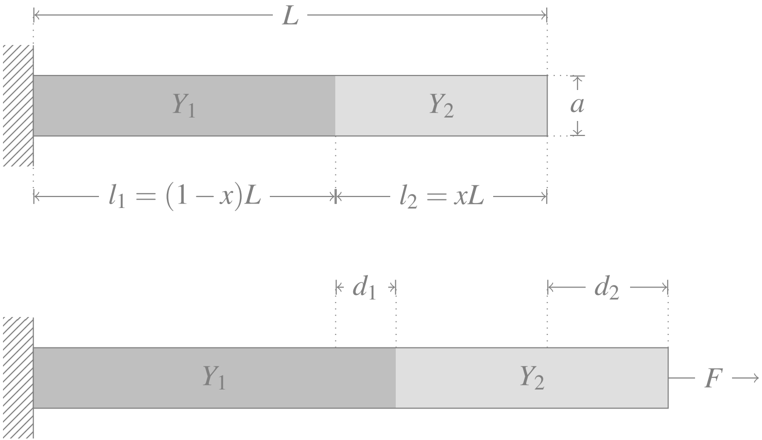
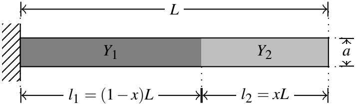
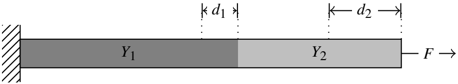


Goal Maximize x under the constraint that $d_2 < D$.

Two-component massless rod, tensile load: FE analysis





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Two-component massless rod, tensile load: FE analysis

FE analysis 3 nodes, boundary conditions

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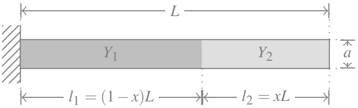

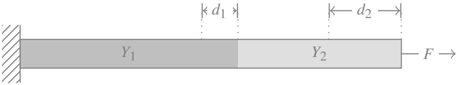

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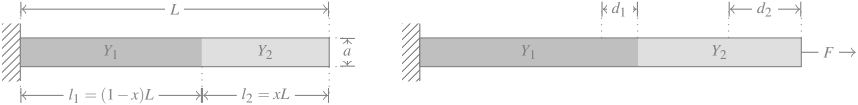

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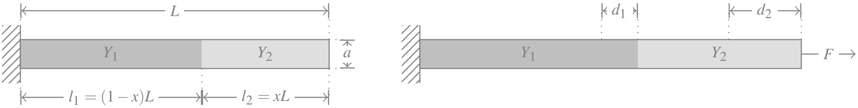

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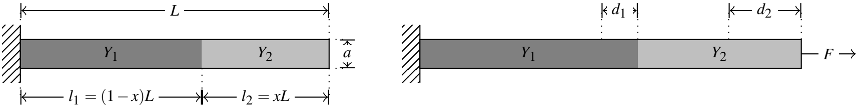
Two-component rod, tensile load: design optimization



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Two-component rod, tensile load: design optimization

Precisely known elastic moduli Y_1 and Y_2 This problem is

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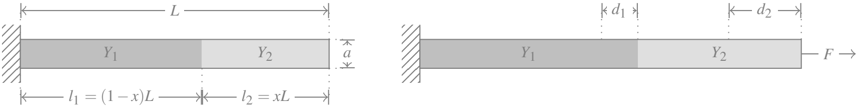

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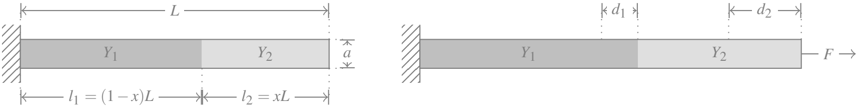

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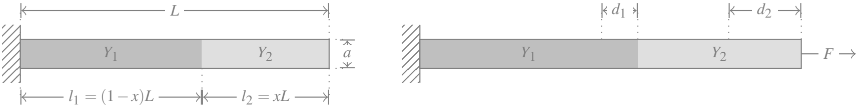
Approach:

- reformulate as a well-posed decision problem;
- solve the decision problem, i.e.,
  derive a classical constrained optimization problem.



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A constrained optimization problem under uncertainty

Goal Maximize $f(x)$ under the constraint that xRY.

- x optimization variable (values in $\mathscr{X}$)
- f objective function (from $\mathscr{X}$ to $\mathbb{R}$)
- Y random variable (realizations y in $\mathscr{Y}$)
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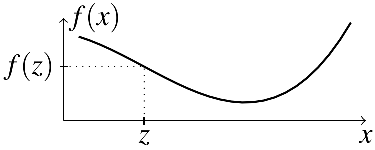
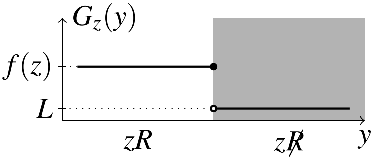
Decision problem Find the optimal decisions x:

- associate a utility function with every decision z:

$$G_z(y) = \begin{cases} f(z), & zRy, \\ L, & z\not{R}y, \end{cases}$$

with penalty value $L < \inf f$;





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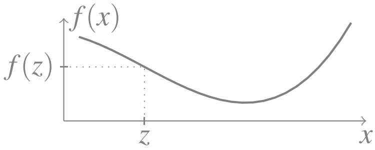

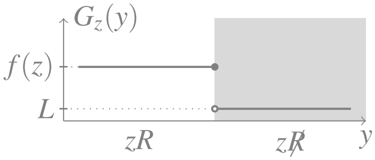

- choose an optimality criterion, e.g., maximinity, maximality.

Uncertainty models & Optimality criteria

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Uncertainty models & Optimality criteria

Lower and upper expectation With (almost) all uncertainty models correspond lower and upper expectation operators ($\underline{E}$ and $\overline{E}$).

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Research results decision problem solutions for some combinations of uncertainty model and optimality criterion; more are on the way.

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- No reduction in the computational complexity;
  one faces
  - an optimization problem to find the uncertainty-independent constraints (cf. doing an FE analysis under uncertainty),
  - the resulting classical constrained optimization problem.