Immediate prediction under exchangeability & representation insensitivity

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1 The setting

Systems of predictive lower previsions: The inferences and predictions of a predictive system family may depend on the actual choice of the model. So we let our subject consider predictive families for all conceivable choices of Φ. We collect families in a system \( \mathcal{C} \) of predictive lower previsions:

\[
\mathcal{C} = \{ \phi \mid \phi \text{ is a finite and non-empty set of } \}
\]

Immediate prediction: The subject in some way uses zero or more observations \( X_1, \ldots, X_n \) made previously (so \( X_1 \ldots, X_n \) belongs to \([0,1]\ldots, N))\) to predict, or make inferences about, the value of the next observation \( X_{n+1} \).

Families of predictive lower previsions: The subject can determine, beforehand, a finite and non-empty set \( \mathcal{C} \) of possible values, or categories, for the random variable. For each \( n \) and each sequence \( x = (x_1, \ldots, x_n) \in \mathcal{C} \), she can give a predictive lower prevision \( \mathcal{C}(x) \) for \( X_n \), given the values \( X_1, \ldots, X_n \). It is defined on the set of all gambles \( f \) on \( \mathcal{X} \).

\[
\mathcal{C}(x) = \{ \phi \mid \phi \text{ is a finite and non-empty set of } \}
\]

An \( \mathcal{C} \)-family \( \mathcal{C} \) of predictive lower previsions is the set formed for all possible observations:

\[
\mathcal{C} = \{ \phi \mid \phi \text{ is a finite and non-empty set of } \}
\]

Precise predictive families are those that only contain precise lower previsions. Predictive systems can be partially ordered: the system \( \Phi \) is more conservative than the system \( \Phi' \), if each predictive lower prevision \( \mathcal{C}(x) \) in \( \Phi' \) is point-wise dominated by the corresponding predictive lower prevision \( \mathcal{C}(x) \) in \( \Phi \).

\[
\mathcal{C}(x) = \{ \phi \mid \phi \text{ is a finite and non-empty set of } \}
\]

A collection \( \{ \mathcal{C}(x) \} \) of predictive lower previsions may have an infimum with respect to this partial order. Whenever it exists, this infimum system \( \Phi \) can be seen as a lower envelope: the lower probability of its predictive lower previsions \( \mathcal{C}(x) \) is defined as the lower envelope \( \inf \{ \mathcal{C}(x) \mid x \in \mathcal{X} \} \) of the predictive lower previsions in the predictive system \( \mathcal{C} \).

Selected references

