Imprecise probabilistic models for inference in exponential families

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Sketch of the context

- There is some stochastic process generating samples.
- Make inferences about 'things' depending on this process.
  - Parametric: depending on the value of its parameters.
  - Predictive: depending on the next sample(s).
- Inferences are typically expressed using
  - probabilities of events, or
  - previsions of gambles.
- The sample size is possibly small, so instead use
  - lower & upper probabilities of events, or
  - lower & upper previsions of gambles.
- A classical inference model structure is used:
  - Choose a prior model, and
  - update it with the sample data using (generalized) Bayes's rule: "prior and likelihood combine into a posterior".
  - Generate inferences with the posterior.

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Exponential family sampling models

- Stochastic processes we look at:

  exponential family sampling models.

- Exponential families:

  Normal, Poisson, Exponential, Bernoulli,. . .

- Typical exponential family form:
  - For a sequence $\boldsymbol{x}$ of m samples,

$$\mathrm{Ef}_{\psi}(\boldsymbol{x}) = \mathrm{a}(\boldsymbol{x}) \exp^{m}\left(\langle \psi, \overline{\tau}(\boldsymbol{x}) \rangle - \mathrm{b}(\psi)\right).$$

- Other concepts:
  - $\mathrm{SEf}_{\boldsymbol{x}}(\psi) = \mathrm{Ef}_{\psi}(\boldsymbol{x})$,
  - Sufficient statistic $(m, \overline{\tau}(\boldsymbol{x}))$, and
  - The likelihood function

$$\mathrm{LEf}_{m,\overline{\tau}(\boldsymbol{x})}(\psi) = \exp^{m}\left(\langle \psi, \overline{\tau}(\boldsymbol{x}) \rangle - \mathrm{b}(\psi)\right).$$

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The foundations: conjugate & predictive distributions

- Basis of the prior & posterior models:

  conjugate families of distributions.

- Typical conjugate family form:

$$\mathrm{CEf}_{n,y}(\psi) = \mathrm{c}(n,y) \exp^{n}\left(\langle \psi, y \rangle - \mathrm{b}(\psi)\right).$$

- Updating is done using Bayes's rule:

$$\mathrm{CEf}_{n_0,y_0} \mathrm{LEf}_{m,\overline{\tau}(\boldsymbol{x})} \propto \mathrm{CEf}_{n_0+m, \frac{n_0 y_0 + m\overline{\tau}(\boldsymbol{x})}{n_0+m}}.$$

- $\mathrm{CEf}_{n,y}$ is the basis for the parametric inference models.
- The predictive family of distributions is derived from the conjugate family:

$$\mathrm{PEf}_{n,y}(\boldsymbol{x}) = \int_{\Psi} \mathrm{CEf}_{n,y} \mathrm{SEf}_{\boldsymbol{x}} = \frac{\mathrm{c}(n,y)\mathrm{a}(\boldsymbol{x})}{\mathrm{c}\left(n+m, \frac{ny+m\overline{\tau}(\boldsymbol{x})}{n+m}\right)}.$$

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The 'precise' inference models I

- The inference models are linear previsions defined using a distribution:
  - The conjugate distribution is used for the parametric model,

$$P_{\mathrm{C}}(f \mid n, y) = \int_{\Psi} f \, \mathrm{CEf}_{n,y},$$

  - the predictive distribution is used for the predictive model,

$$P_{\mathrm{P}}(g \mid n, y) = \int_{\mathcal{X}_m^*} g \, \mathrm{CEf}_{n,y}.$$

- The prevision of particular gambles is easy to calculate:
  - Considering that $\nabla_{\psi} \mathrm{b}(\psi) = \int_{\mathcal{X}_m^*} \overline{\tau} \mathrm{Ef}_{\psi}$, it is a nice result for the parametric model that

$$P_{\mathrm{C}}(\nabla \mathrm{b} \mid n, y) = y.$$

  - An analogous result holds for the predictive model:

$$P_{\mathrm{P}}(\overline{\tau} \mid n, y) = y.$$

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The 'precise' inference models II

- Problem: how to choose n_0 and y_0?
- Effect on the parameters of updating with a sequence of m samples $\boldsymbol{x}$:

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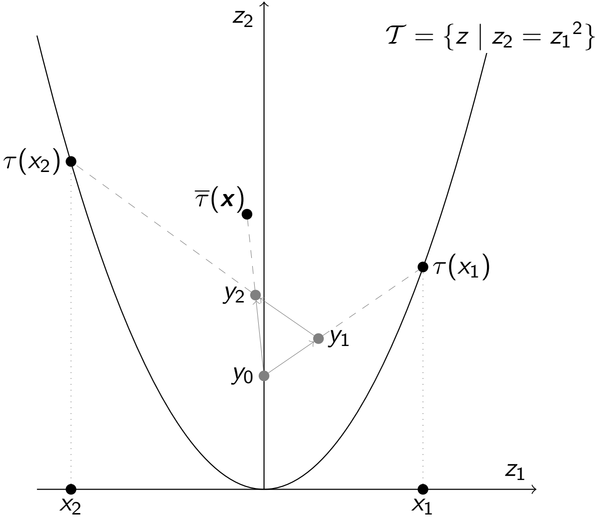
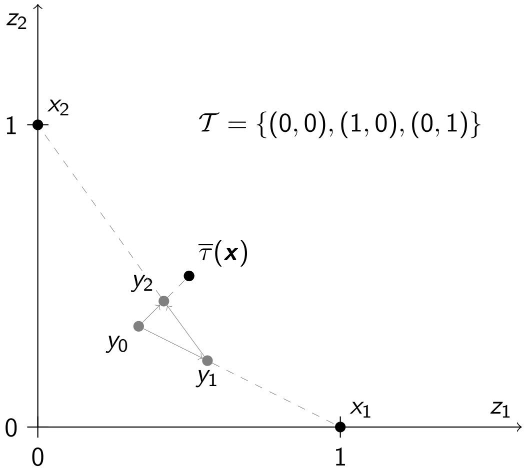
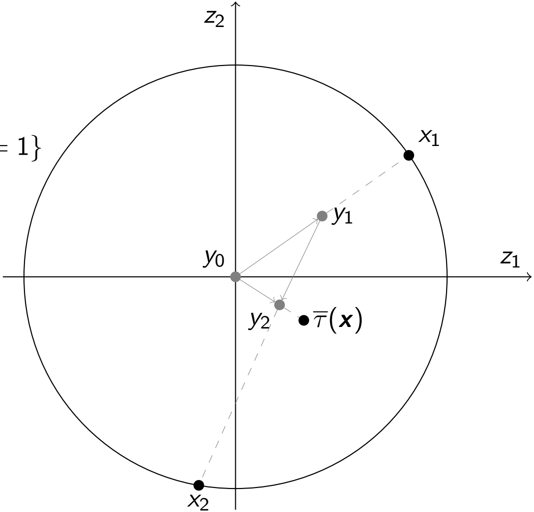


Updating the ‘precise’ inference models: Bernoulli



Updating the 'precise' inference models: von Mises

$$\mathcal{T} = \mathbb{S}_2 = \{z \mid \|z\| = 1\}$$



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- The inference models are lower previsions defined using sets of distributions:
  - Sets of conjugate distribution for the parametric model,
  $$\underline{P}_{\mathrm{C}}(f \mid n, \mathcal{Y}) = \inf_{y \in \mathcal{Y}} P_{\mathrm{C}}(f \mid n, y),$$
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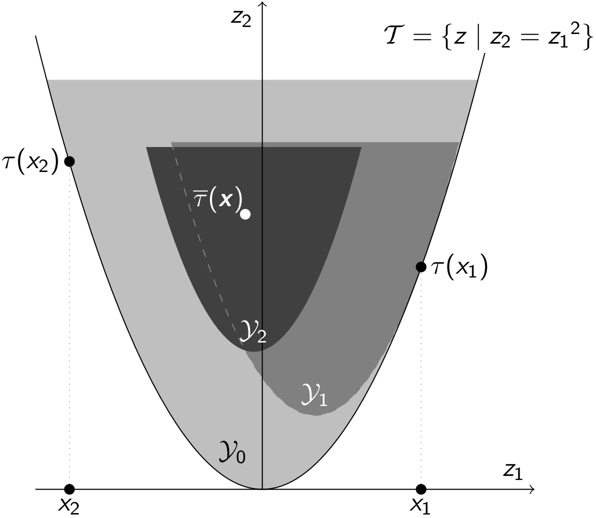

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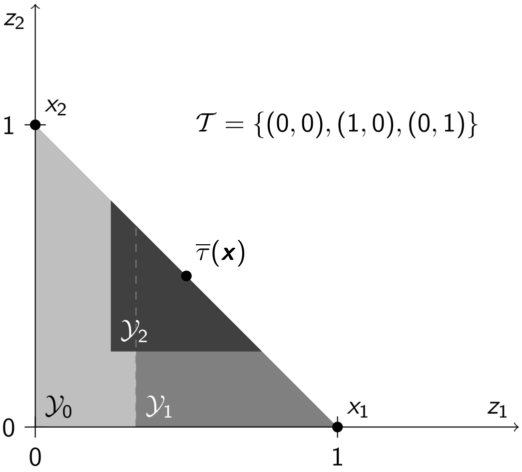

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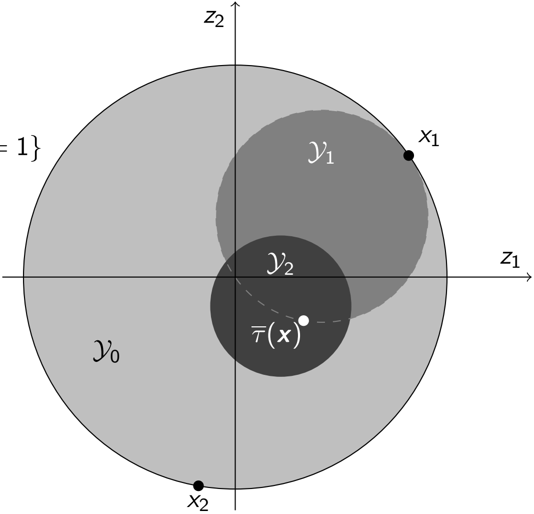

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- Calculate lower and upper cumulative distribution functions.
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- Combine inference models for multiple stochastic processes:
  - Combining independent marginals, or
  - combining marginal and conditional models.

Summary

- For *all* exponential family sampling models, imprecise probabilistic parametric and predictive inference models can be defined.
- Updating the models with sample data consist of a *straightforward* modification of the model parameters.
- Some inferences can be obtained very easily.

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  - What other inferences can be calculated?
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