ERRATUM TO:
FINITELY ADDITIVE EXTENSIONS OF DISTRIBUTION FUNCTIONS AND MOMENT SEQUENCES: THE COHERENT LOWER PREVISION APPROACH

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There is a mistake in the proofs of Lemmas 11 and 12 in Section 5 of our paper [2]. It is wrongly assumed there that for every gamble h on [0,1] and every $t \in [0,1]$, $\underline{\text{osc}}(h) > t$ coincides with $\text{int}(\{h > t\})$. To see that we indeed have the inclusion, note that if $\underline{\text{osc}}(h)(x) > t$, then there is some open set B that includes x such that $\inf_{y \in B} f(y) > t$, so B is included in $\{f > t\}$ and therefore x belongs to the interior of $\{f > t\}$. However, the inclusion can be strict, as the following example shows:

Example 1. *Let f be the gamble given by*

$$f(x) = \begin{cases} 2 & \text{if } x = 0.5 \\ 1 + |x - 0.5| & \text{otherwise.} \end{cases}$$

Then $\{f > 1\} = [0,1]$, *whence* $\text{int}(\{f > 1\}) = [0,1]$. *However, for every open set B that includes* $x = .5$, *we see that* $\inf_{y \in B} f(y) = 1$, *and therefore* $\underline{\text{osc}}(f)(x) = 1$. *As a consequence,* $\{\underline{\text{osc}}(f) > 1\}$ *is a strict subset of* $\text{int}(\{f > 1\})$.

Nevertheless, both Lemmas 11 and 12 are correct, as we show next:

Corrected proof of Lemma 11. Let us show first of all that $\underline{\text{osc}}(h)$ is lower semicontinuous. Consider $t \in \mathbb{R}$ and $x \in \{\underline{\text{osc}}(h) > t\}$. Then there is some open set B including x such that $\inf_{y \in B} f(y) > t$. As a consequence, $B \subseteq \{\underline{\text{osc}}(h) > t\}$, and this implies that $\{\underline{\text{osc}}(h) > t\}$ is open. Since this holds for all real t, we deduce that $\underline{\text{osc}}(h)$ is lower semicontinuous. It also follows from its definition that $\underline{\text{osc}}(h) \leq h$.

Next, consider any lower semicontinuous mapping g that is dominated by h. Then for any real number t and any x in the open set $\{g > t\}$, it holds that $\underline{\text{osc}}(g)(x) \geq t$, whence also $\underline{\text{osc}}(h)(x) \geq \underline{\text{osc}}(g)(x) \geq t$. As a consequence, $\{g > t\} \subseteq \{\underline{\text{osc}}(h) \geq t\}$ for all real t, and this implies that $g \leq \underline{\text{osc}}(h)$. We conclude that $\underline{\text{osc}}(h)$ is the greatest lower semicontinuous gamble that is dominated by h. □

In general, we have the following chain of inclusions

$$\{\underline{\text{osc}}(h) > t\} \subseteq \text{int}(\{h > t\}) \subseteq \text{int}(\{h \geq t\}) \subseteq \{\underline{\text{osc}}(h) \geq t\}.$$

To see that the last inclusion holds, observe that if $x \in \text{int}(\{h \geq t\})$, then there is some open set B that includes x such that $h(y) \geq t$ for all $y \in B$, and therefore $\underline{\text{osc}}(h)(x) \geq \inf_{y \in B} h(y) \geq t$.

Using this chain of inequalities, we can also establish Lemma 12:

Corrected proof of Lemma 12. For any gamble h on $[0,1]$ and any $d \in [0,1]$, it holds that

$$\int_{\inf h}^{\sup h} I_{\{\underline{\text{osc}}(h) > t\}}(d)dt \leq \int_{\inf h}^{\sup h} I_{int\{h \geq t\}}(d)dt \leq \int_{\inf h}^{\sup h} I_{\{\underline{\text{osc}}(h) \geq t\}}(d)dt,$$

and both the first and third integrals are equal to $\underline{\text{osc}}_d(h) - \inf h$. □

The correct arguments can also be found in a further paper [1].

REFERENCES

[1] Gert de Cooman and Enrique Miranda. The F. Riesz Representation Theorem and finite additivity. In Didier Dubois, María Asunción Lubiano, Henri Prade, María Ángeles Gil, Przemysław Grzegorzewski, and Olgierd Hryniewicz, editors, *Soft Methods for Handling Variability and Imprecision (Proceedings of SMPS 2008)*, pages 243–252. Springer, 2008.

[2] Enrique Miranda, Gert de Cooman, and Erik Quaeghebeur. Finitely additive extensions of distribution functions and moment sequences: The coherent lower prevision approach. *International Journal of Approximate Reasoning*, 48:132–155, 2008.

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