Imprecise Probability in Statistical Modelling: A Critical Review

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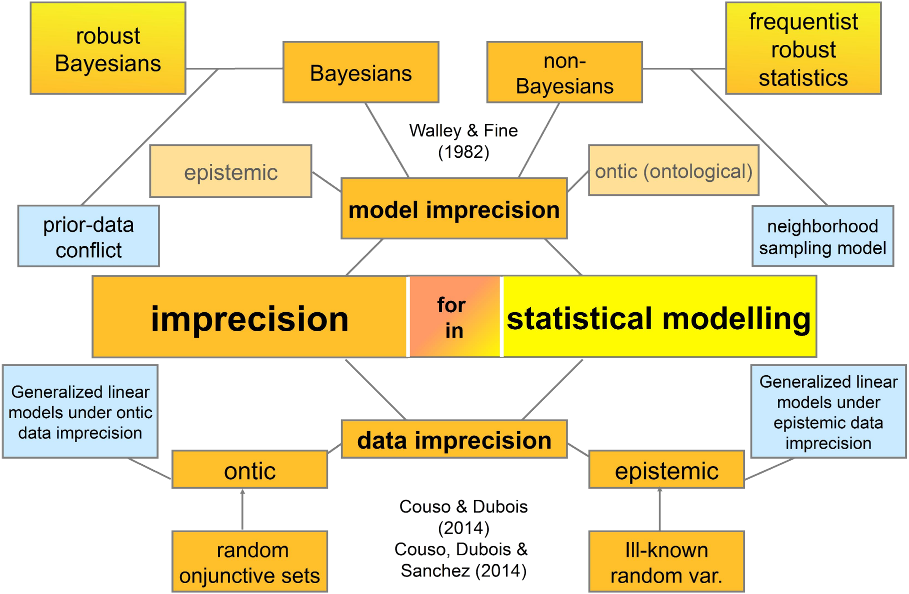
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Imprecision in statistics

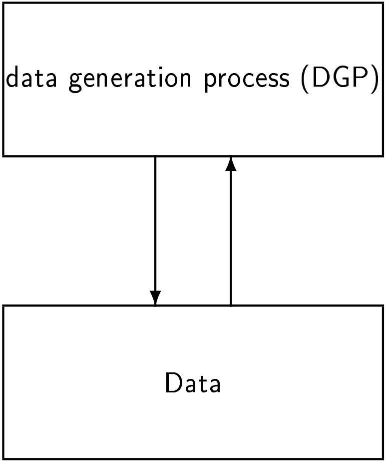
– hide/neglect imprecision!
– model imprecision away!

!! take imprecision into account in a reliable way!

!! imprecision as a modelling tool

1. Introduction

Statistics



Two kinds of imprecision

- **data imprecision:** imprecise observations, data are subsets of the intended sample space
  - imprecise observations of something precise $\rightarrow$ epistemic
  - precise observations of something imprecise $\overset{\approx}{\rightarrow}$ ontic

  Couso & Dubois (2014, IJAR), Couso, Dubois & Sánchez (2014, Springer)
- **model imprecision:** imprecise probability models

$$P(Data||Parameter),$$

  maybe also $P(Parameter)$
- **set-valued approaches:** take **sets** of values/probability distributions as the basic entity

On the power of IP in statistical modelling

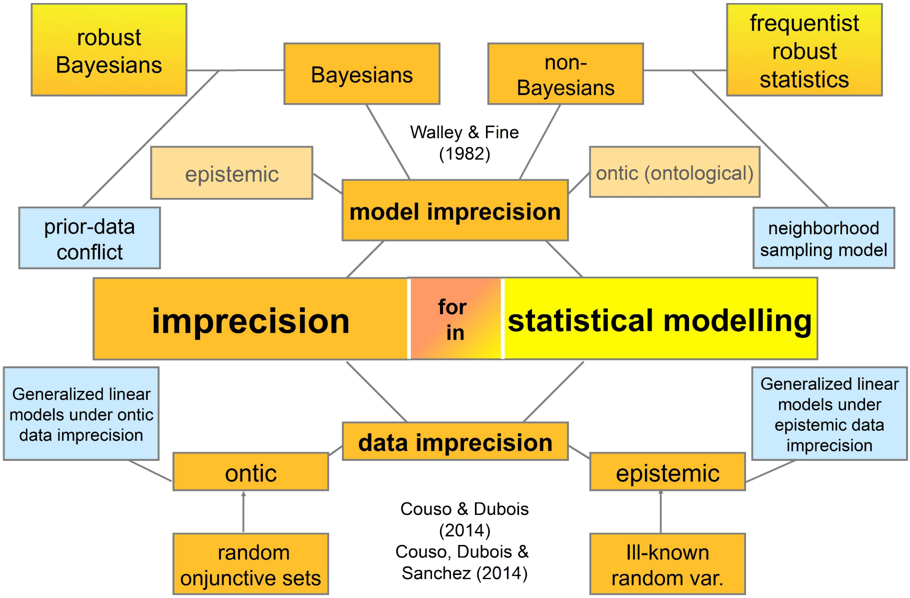
defensive point of view

- IP protects against the potentially disastrous consequences of applying standard procedures under violated assumptions $\rightarrow$ robustness in:
- frequentist and
- Bayesian settings

On the power of IP in statistical modelling

offensive point of view
IP is a most powerful methodology, allowing for

- separation of variability (variance) from indeterminism
- active modelling of ignorance
- active modelling of conflicting/surprising information
- active use of weak knowledge that can not be used in the traditional setting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The mantra of statistical modelling

Box & Draper (1987, Empirical Model Building and Response Surfaces, p. 424)

- "Essentially, all models are wrong,

The mantra of statistical modelling

Box & Draper (1987, Empirical Model Building and Response Surfaces, p. 424)

- “Essentially, all models are wrong,
- but some of them are useful”,

The mantra of statistical modelling

Box & Draper (1987, Empirical Model Building and Response Surfaces, p. 424)

- "Essentially, all models are wrong,
- but some of them are useful",
- and sometimes dangerous

Assumptions may matter!

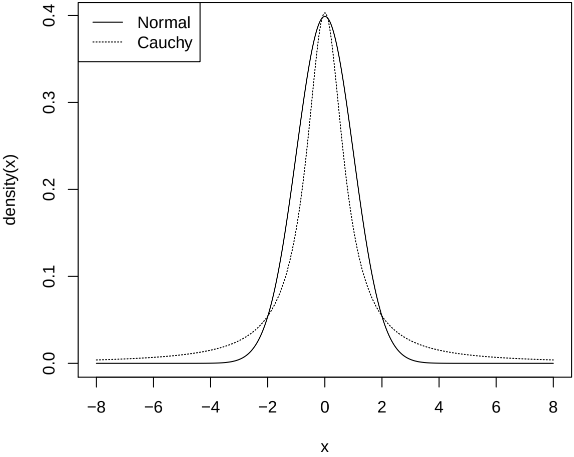


Figure: Densities of the Normal$(0, 1)$ and the Cauchy$(0, 0.79)$ distribution.

Assumptions may matter!

Consider sample mean $\overline{X}$.

- if $X_1, \ldots, X_n \sim N(\mu, 1)$ (normally distributed), then

$$\bar{X} \sim N(\mu, \frac{1}{n})$$

  Learning from the sample, with increasing sample size variance of $\overline{X}$ decreases.

- if $X_1, \ldots, X_n \sim \mathcal{C}(\mu, 1)$ (Cauchy-distributed), then

$$\overline{X} \sim C(\mu, 1)$$

  Distribution does not depend on n, no learning via sample mean possible

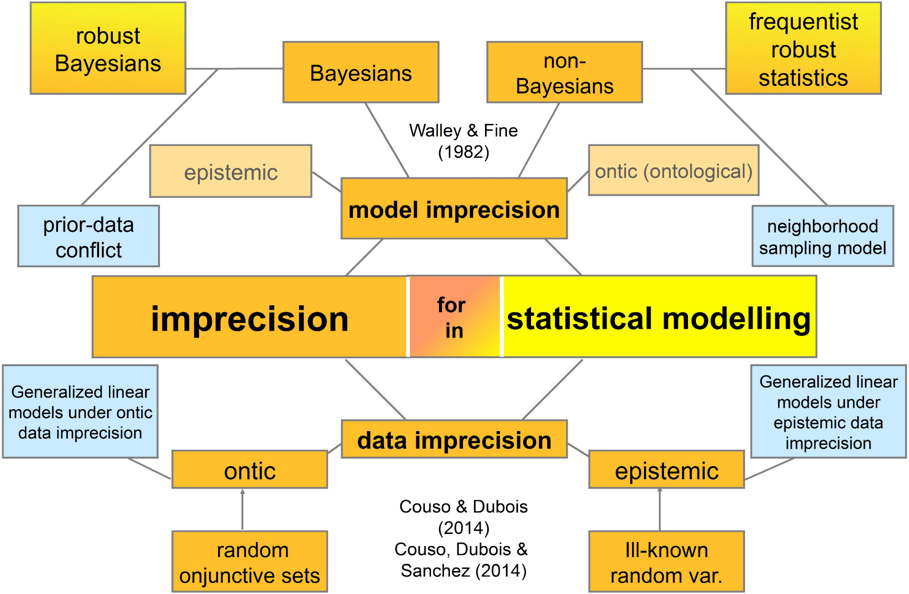
Assumptions may matter! robustness

- many optimal procedures show very bad properties under minimal deviations from the ideal model
- instead of $f(x||\vartheta)$: model "approximately $f(x||\vartheta)$ ", i.e. consider all distribution "close to $f(x||\vartheta)$ "
  $\rightarrow$ neighbourhood models

Huber & Strassen approach

Huber & Strassen (1973, AnnStat): globally least favorable pairs for optimal Neyman-Person testing between two-monotone surveyed, e.g., in Augustin, Walter & Coolen (2014, Intro IP, Wiley)

* applicable to most neighborhood models of precise probabilities
* extension to neighborhood models of many IP models
* construction procedures
* going beyond two-monotonicity
  - parametrically constructed models
  - locally least favorable pairs



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Bayesian inference with sets of priors I: ignorance

- so-called 'noninformative priors' do contain information
- consider set of all (non-degenerated) distributions instead, e.g., Walley (1996, JRSSB), Benavoli & Zaffalon (2012, JSPI)

Bayesian inference with sets of priors II: prior-data conflict

- Bayesian models are understood to express prior knowledge (or to "borrow strength")
- what happens when this prior konwledge is wrong?
- example: $X_1, \ldots, X_n$ i.i.d data, $X_i \sim \mathcal{N}(\mu, \sigma_0^2)$
  conjugated prior: $\mu \sim \mathcal{N}(\nu, \varrho^2)$ then

$$
\begin{aligned}
\nu' &= \frac{\bar{x}\rho^2 + \nu \cdot \frac{\sigma^2}{n}}{\rho^2 + \frac{\sigma^2}{n}} \\
\rho^{2'} &= \frac{\rho^2 \cdot \frac{\sigma^2}{n}}{\rho^2 + \frac{\sigma^2}{n}}
\end{aligned}
$$

Bayesian inference with sets of priors II: prior-data conflict

- let, for sake of simplicity, $\varrho^2 = \frac{\sigma^2}{n}$, then

$$\hat{\mu} = \nu' = \frac{\bar{x} + \nu}{2}$$

and

$$\varrho^{2'} = \frac{\varrho^4}{2\varrho^2} = \frac{\varrho^2}{2}.$$

- then, e.g.,

$$\bar{x} = 0.9 \text{ and } \nu = 1.1$$

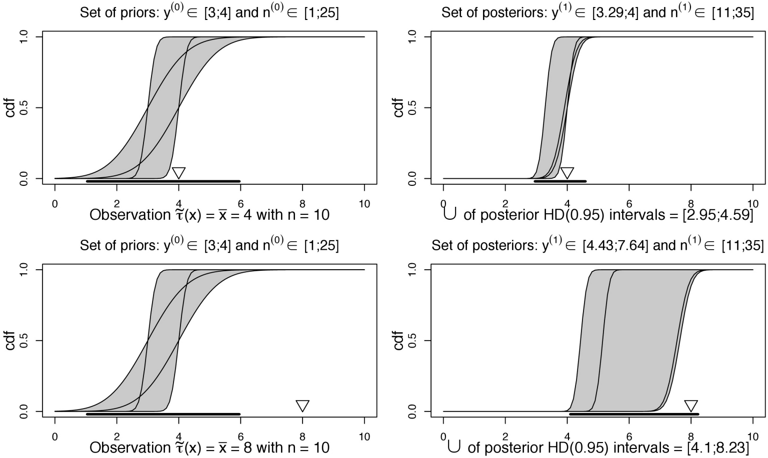
and

$$\bar{x} = -100 \text{ and } \nu = 102$$

lead to the same distribution (equal mean and variance)
- general effect for canonical exponential families
- much more intuitive behaviour when prior parameters are imprecise, e.g., are interval-valued

Bayesian inference with sets of priors II: prior-data conflict

Source: Walter & Augustin (2009, JStTheorPract, p. 268)



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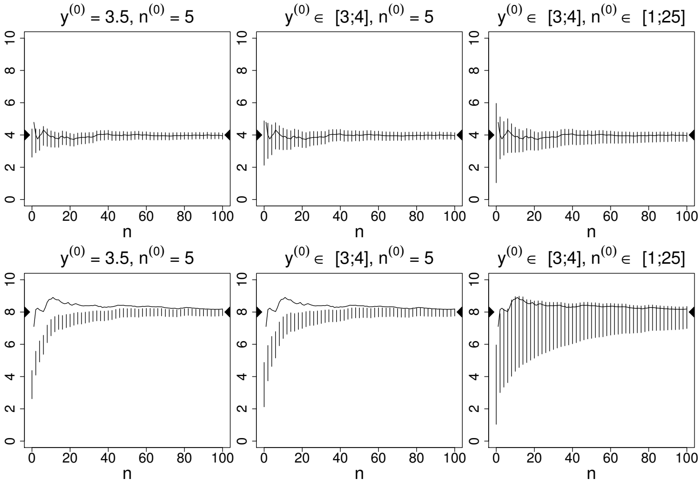

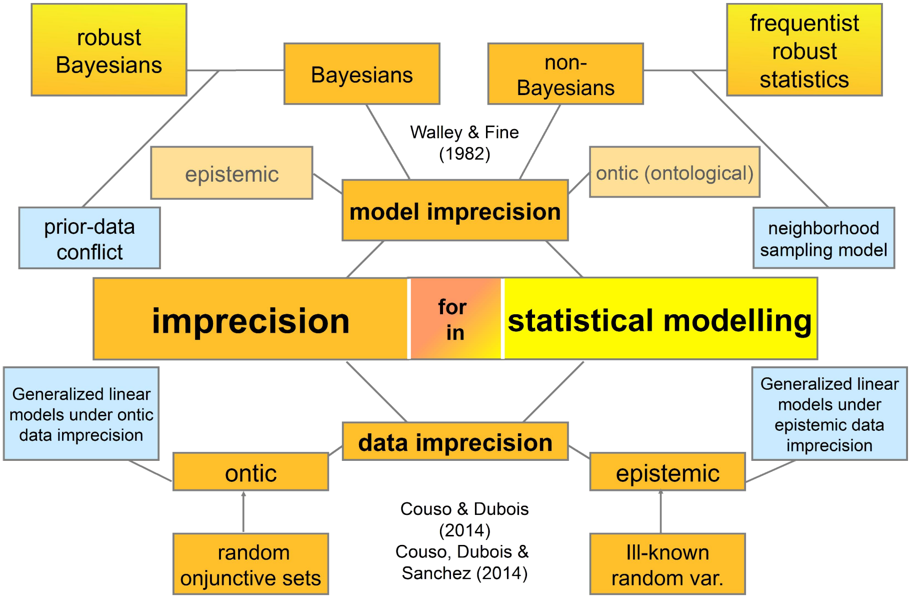
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Ontic imprecision: example

Plass, Fink, Schöning & Augustin (2015, ISIPTA)

- pre-election study (GLES 2013: German Longitudinal Election Study)
- a considerable amount of voters is still undecided, but mainly only between two or three parties
- these voters constitute different subgroups of there own with specific characteristics (which have to be neglected in the traditional analysis)
- here, for the moment, NO forecast aimed at, instead analysis of individual preferences as they are

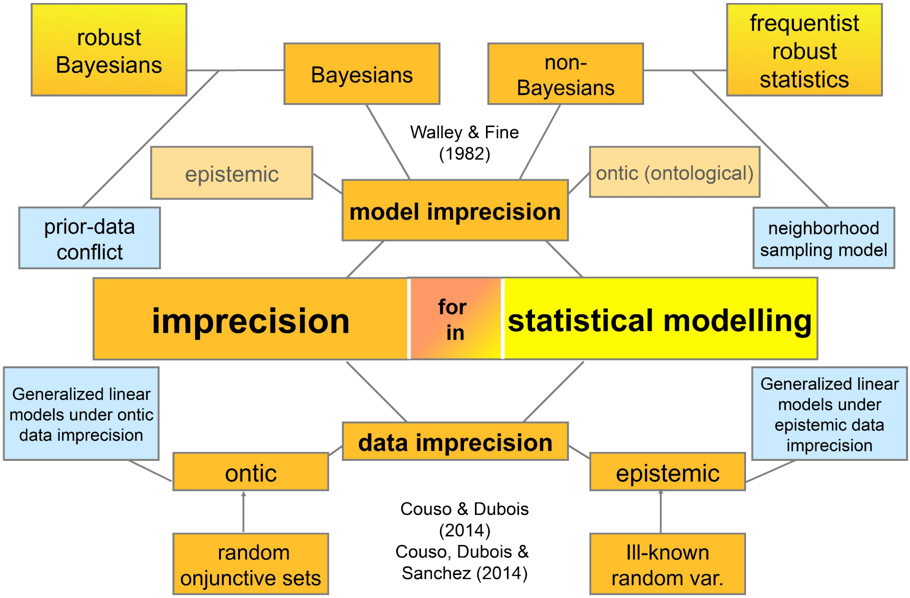
Ontic imprecision: modelling idea

- "precise observations of something imprecise"
- modelled by random conjunctive sets
- change sample space $\mathcal{S} = \{CD, SPD, Green, Left, \ldots\}$ into $\mathcal{S}^* \subset \mathcal{P}(\mathcal{S})$
- oberservations are precise observations in $\mathcal{S}^*$ and can be treated like tradtional categorical data
- whole statistical modelling framework can be applied, here logistic regression
- for each non-empty element of $\mathcal{S}^*$ vector of regression coefficients

Ontic imprecision: example, Plass et al (2015, Table 4)

Coefficient	ontic		classical
	CD	G:S	CD
intercept	0.33	−1.41 **	−0.12
rel.christ	0.37 **	−0.25	0.52 ***
info.tv	−0.02	−0.32	0.25
info.np	−0.12	−1.69 **	0.13

Table 4: Comparison of results (first vote).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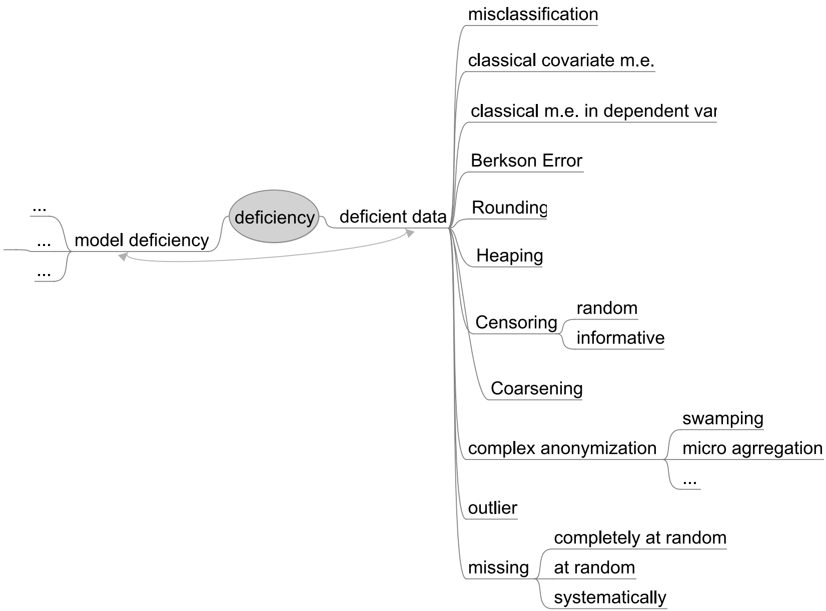
Epistemic data imprecision

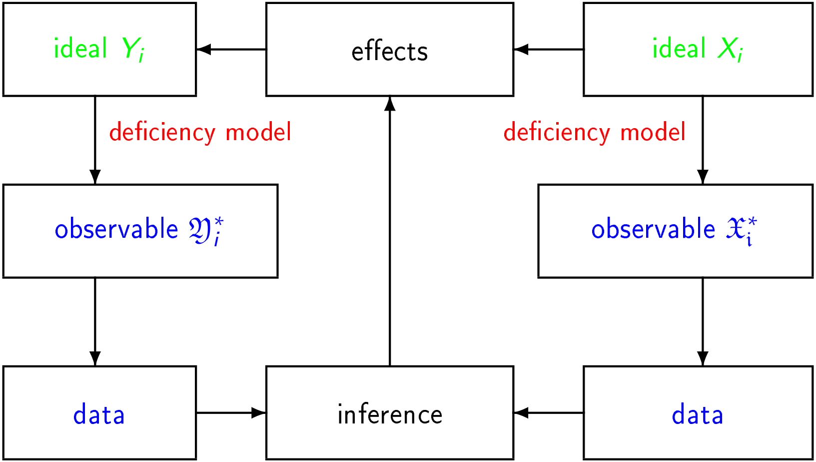
imprecise observations of something precise

- missing data (refusals, treatment design)
- data protection
- data merging with partially overlapping categories
- secondary data analysis
- forecasts derived from set-valued (ontic) observations
- refined responses of primary refusals, typically coarsening/missing not at random

Spinney of deficiencies



The two-layers perspective



Traditional treatment of deficiencies

- model the deficiency process!
- characterize situations where the deficiency may be ignored or when one can correct for it!
- but typically very restrictive – often untestable – asumptions needed to ensure identifiability = precise solution

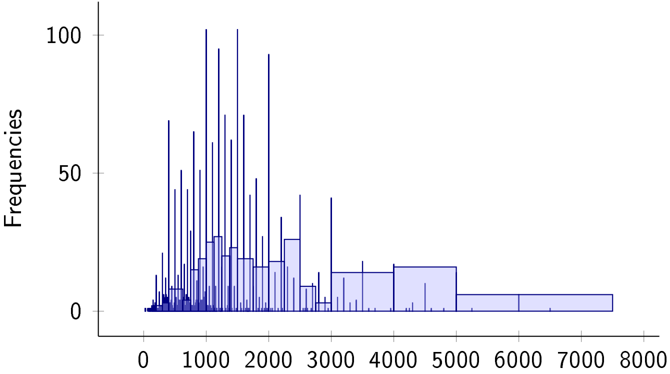
Traditional treatment of deficiencies

For instance, in measurement error models ("classical model of testing theory"):

- measurement error model must be known precisely
  - type of error, especially assumptions on (conditional) independence
    - independence of true value
    - independence of other covariates
    - independence of other measurements
  - type of error distribution
  - moments of error distribution
- validation studies typically not available

Interval data: example

German General Social Survey (ALLBUS) 2010:
2827 observations in total, approx. 2000 report personal income (30% missing). An additional 10% report only income brackets.



Interval data: example

1. we see *heaping* at 1000 €, 2000 €, ..., less so at 500 €, 1500 €, ...
2. both heaping and grouping depend on the amount of income reported.
3. missingness (some 20% of the data) might as well depend on the amount of income.

Interval data: example

1. we see *heaping* at 1000 €, 2000 €, ..., less so at 500 €, 1500 €, ...
2. both heaping and grouping depend on the amount of income reported.
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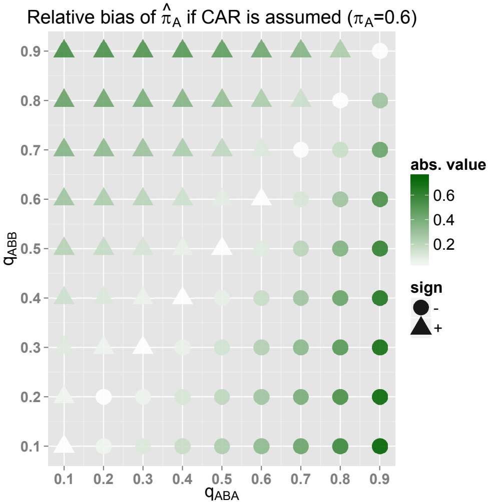
Consequences:

1. missingness, grouping, and heaping can often be represented by intervals.
2. missingness, grouping, and heaping will rarely conform to the assumption of "coarsening at random" (CAR).
3. missingness, grouping, and heaping add an additional type of uncertainty apart from classical statistical uncertainty. This uncertainty can't be decreased by sampling more data.

Wrongly assuming CAR (binary data)

Source: Plass, Augustin, Cattaneo, Schollmeyer (2015, ISIPTA)



Manski's Law of Decreasing Credibility

Reliability !? Credibility ?

"The credibility of inference decreases with the strength of the assumptions maintained." (Manski (2003, p. 1))

Reliable inference instead of overprecision!!

Consequences to be drawn from the Law of Decreasing Credibility:

- adding untenable assumptions to produce precise solution may distroy credibility of statistical analysis, and therefore its relevance for the subject matter questions.
- make *realistic* assumptions and let the data speak for themselves!
- the results may be imprecise, but are more reliable
- the extent of imprecision is related to the data quality!
- as a welcome by-product: clarification of the implication of certain assumptions
- often still sufficient to answer subjective matter question

Much IP work on epistemic date impprecision, e.g.,

- de Cooman & Zaffalon (2004, AI), Zaffalon & Miranda (2009, JAIR)
- Utkin & Augustin (2007, IJAR), Troffaes & Coolen (2009, IJAR)
- Utkin & Coolen (2011, ISIPTA)
- Cattaneo & Wiencierz (2012, IJAR)
- Schollmeyer & Augustin (2015, IJAR)
- Denoeux (2014, IJAR)

Relation to work in econometrics, biometrics and engineering

- partial identification: e.g., Manski (2003, Springer), Tamer (2010, Annu Rev Econ)
- systematic sensitivity analysis: e.g., Vansteelandt, Goetghebeur, Kenword, Molenberghs (2006, Stat. Sinica)
- reliable computing, interval computation: e.g., Ferson et al. (2007, Sandra TR), Nguyen, Kreinovich, Wu & Xiang (2011, Springer)

Recent likelihood approach

Plass, Augustin, Cattaneo, Schollmeyer (2015, ISIPTA)

- utilize invariance of likelihood under paramter-transformation
- observable part: set-valued observations, parameter ϑ, maximum likelihood estimator $\hat{\vartheta}$
- latent part: parameter of interest γ
- related via observation model: expressed by mapping Φ
- set-valued maximum likelihood estimator $\hat{\Gamma} = \left\{\gamma | \Phi(\gamma) = \hat{\vartheta}\right\}$
- application also to some basic logistic regression models

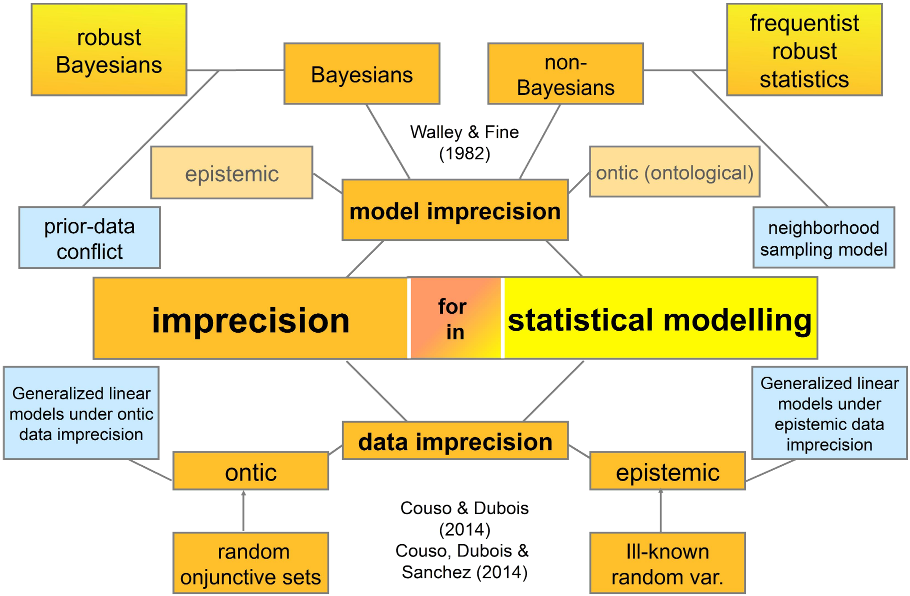
Estimating equations

Generalizing from the linear case, suppose there is a consistent (score-) estimating equation for the ideal model $\{\mathcal{P}_\vartheta \,|\, \vartheta \in \Theta\}$, i.e.:

$$\forall \vartheta \in \Theta : \quad \mathbb{E}_\vartheta(\psi(X, Y; \vartheta)) = 0$$

With interval data, one gets a set of estimating equations, one for each random vector (selection) $(X, Y) \in (\mathfrak{X}, \mathfrak{Y})$:

$$\Psi(\mathfrak{X}, \mathfrak{Y}; \vartheta) := \{\psi(X, Y; \vartheta) \,|\, X \in \mathfrak{X}, Y \in \mathfrak{Y}\}$$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Concluding remarks

- law of decreasing credibility !
- reliable use of information
- set-valued analysis: imprecise data, imprecise models
- imprecise but reliable results; often sufficient!
- natural behaviour of imprecision!
- use this actively in modelling!
- towards a general framework for reliable analysis of non-idealized data!

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defensive point of view

- IP protects against the potentially disastrous consequences of applying standard procedures under violated assumptions $\rightarrow$ robustness in:
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offensive point of view
IP is a most powerful methodology, allowing for

- separation of variability (variance) from indeterminism
- active modelling of ignorance
- active modelling of conflicting/surprising information
- active use of weak knowledge that can not be used in the traditional setting

Future directions

Popularize the defensive point of view

- case studies, illustrating the power of imprecision
- robust procedures for generalized linear models etc.
- cautious data completion for generalized linear models etc.
- (disc. with H. Rieder): for each result complement p-value routinely by stability level: smallest level of contamination where the result is no longer significant

Propagate the offensive view

- case studies, illustrating the power of imprecisions
- separation of variability (variance) from indeterminism
- active modelling of ignorance
- active modelling of conflicting/surprising information
- active use of weak knowledge that can not be used in the traditional setting

Future directions

- statisticians start to think from data
  $\rightarrow$ improve understanding of imprecise *sampling* models
  - imprecise probabilities for the *observables*!
  - generalized sampling theory: imprecise selection probabilities
  - utilize variety of independence concepts (model slight dependence)
  - develop methodology of estimation from imprecise sampling models
- develop simulation techniques for imprecise probabilities
- how to handle regression models?

Future directions

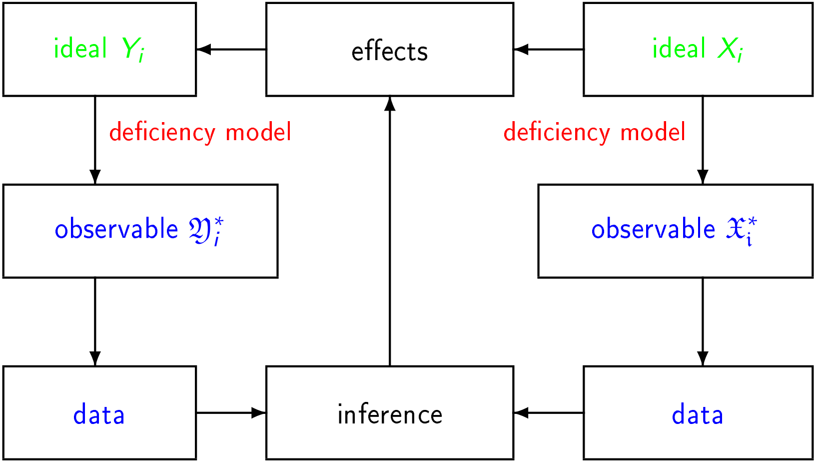
- develop heuristics, "semi-imprecise" methods
  "IP should make life better or easier (or both)" (Frank Coolen)
- develop direct methods
  - leave the necessarily more complicated "set-of traditional model views"
  - direct processing of information (e.g., statistics with desirable gambles?)

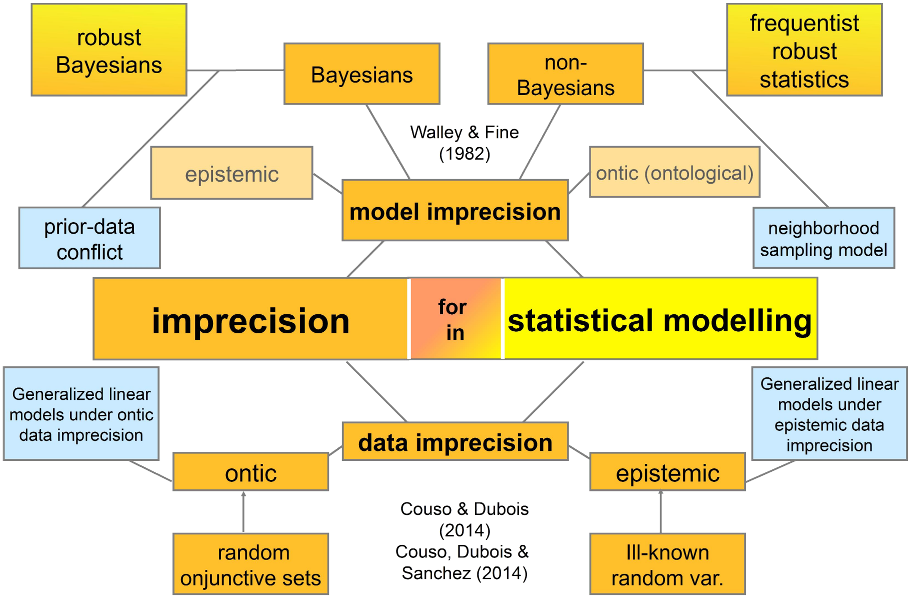
Future directions

- develop a methodology for statistical modelling with sets of models
  - generalized linear models
  - nonparametric regression models → smoothing
  - variable selection
  - realistic measurement error and random effect models
  - importance of unbiased estimation equations

  - evaluation / comparision of models with different level of imprecision

The two-layers perspective

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