Uncertainty Representations and Reasoning

A course on uncertainty modeling beyond probability theory

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Course overview

Goal

Introduction to uncertainty modeling approaches that go beyond classical probability theory

General information

- Elective in TU/e's Data Science & Artificial Intelligence Master program
- Study load: circa 140 hours (5 ECTS)
- First edition in 2022–2023 Q1 (Sep–Nov)
- Students: circa 40, all familiar only with classical probability and statistics

Learning activities

Theory Assignment

- Literature study (report)
- Lectures Practice exercises Poster presentations
- 'Instructions'
 - Explanation course organization
 - Q&A lectures, exercises, assignment

Schedule overview

- Quartile = 8 contact session weeks + 2 exam weeks
- Contact sessions:
- 16 sessions total: 2/week, each 2 blocks of 45 minutes (3 hours/week)
- lectures (18 blocks); instructions (6 blocks); poster presentations (8 blocks)

		Sessions														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Blocks	L	L	L	L	L	L	P	P	L	L L	L	L	L	Р	Р	- 1
	-1	L	-1	L	L		P	P	L	L	L	L		Р	Р	-1

• Exam: 3 hours; resit possibility during exam week of next quartile

Grade composition

50/50 for assignment/exam

Support options

Q&A sessions and lecture breaks

- Online Forum
- Direct message to lecturer

Assignment

Goal

Understand and explain to fellow students how different uncertainty modeling approaches each can deal with a specific application topic

Application topics

• Classification • Clustering • Decision trees • Markov chains • Graphical models

Organization

Setup

Done in pairs

• In parallel to lectures

Deliverables Report

Poster

- Literature to digest
 - Provided: 4–7 papers/topic (1+/approach)

Theory

Other texts also allowed

Report template summary

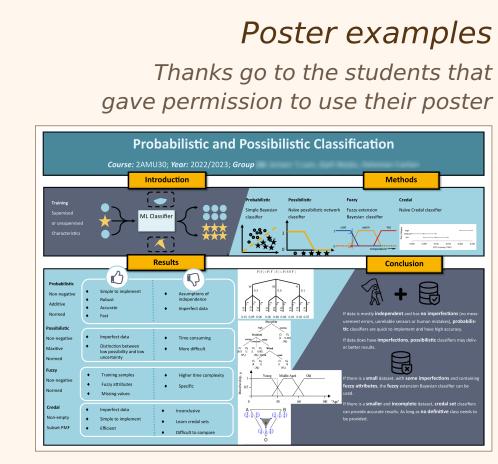
Title Authors Abstract Data+code availability Contributions

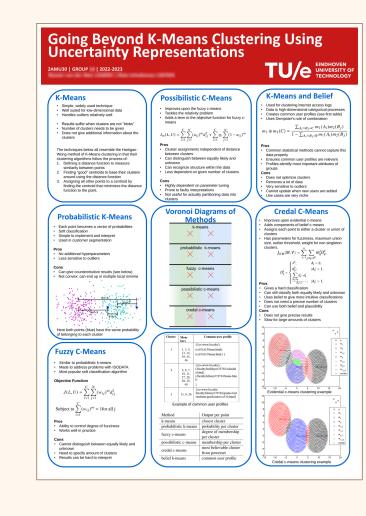
Acknowledgements

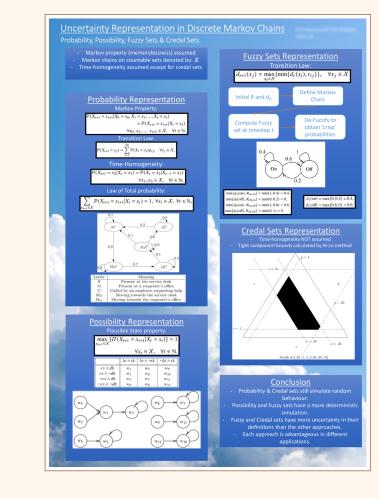
Introduction introduce topic, context, motivation, report overview

Literature discussion conceptual discussion, key contributions, relevance

unified presentation, math, illustration, examples Conclusions advantages, limitations, recommendations







Excellent

Assessment

- Formative: Midterm (session 7–8)
- Peer review by fellow students using rubrics
- Summative: Final (session 14–15)

Scale

Good participation was important (24%)

Problematic Insufficient Sufficient Good

3 points 4 points 0 points 5 points Structure 7.5% presence and quality of structure and structuring elements (sectioning, paragraphs, lists, tables, figures) Clarity 20% degree to which the content is explainable by the reader based Report on a reading (argumentation steps, examples, illustrations) 5% quality of grammar, spelling, and formulation Language Notation 5% introduction and appropriate choice/use of formal notation Mathematics 7.5% presence, clarity, and integration of math expressions in the text 5% degree text is supported by sufficient on-topic references; Referencing completeness of entries

Rubrics summary

Balance 6.5% balance between text/math and illustrations/examples; **Poster** suitability for live explanation (storyline, key takeaways) Presentation 6.5% degree of preparation and capability to answer questions **#Approaches** 13% number and coverage of unc. modeling approaches (aim = 4)

Observations

- Participation was generally enthusiastic
- Most pairs kept to the literature provided
- Pairs often struggled to integrate material from papers using approaches discussed towards the end of the lecture series

Lectures, Exercises, and Exam

Goal

For each of the uncertainty modelling approaches discussed:

- know and understand the foundations & interpretations
- obtain the skills to solve basic inference and decision problems

Lecturing approach

- Theory lectures in classical style
- Illustrative examples mixed in
- Successful opportunities for interaction
- Students were encouraged to interrupt
- Activating questions from lecturer

Lecture topics & Uncertainty modeling approaches

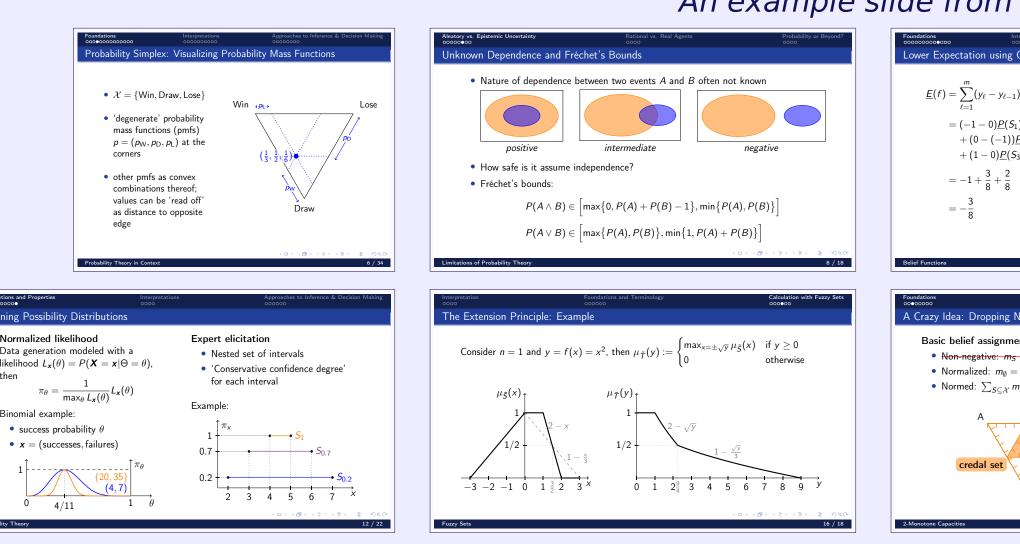
1. Probability (classical)

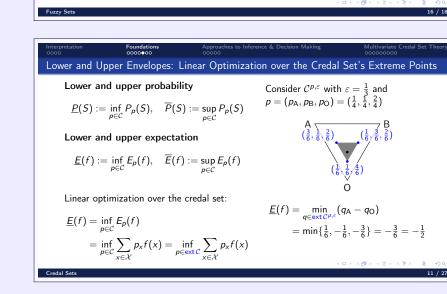
3. Belief functions

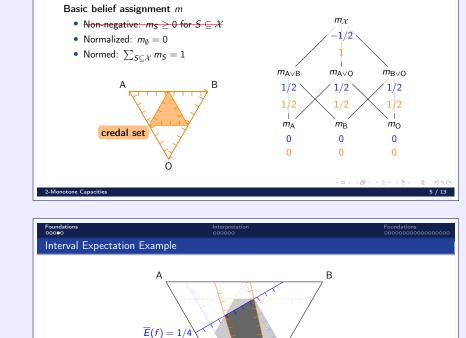
(arguments to go beyond)

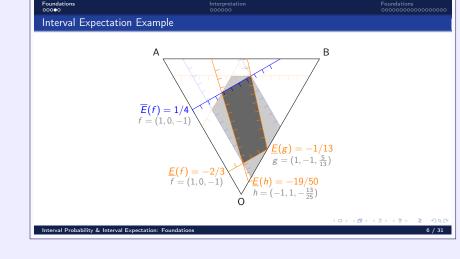
- 2. Limitations of probability
- 4. Possibility 5. Fuzzy sets
- 7. Probability intervals
- 8. Credal sets
- 6. 2-Monotone capacities 9. Interval expectation
 - ('prevision' mentioned)

An example slide from each of the lectures









Much of the actual content was inspired by materials from the SIPTA Schools

Focus areas

onsider two probability intervals (p^1, \overline{p}^1) and (p^2,\overline{p}^2) , how can we combine then

 $\underline{\rho}_{\scriptscriptstyle X} := \min\{\underline{\rho}_{\scriptscriptstyle X}^1,\underline{\rho}_{\scriptscriptstyle X}^2\}, \quad \overline{\rho}_{\scriptscriptstyle X} := \max\{\overline{\rho}_{\scriptscriptstyle X}^1,\overline{\rho}_{\scriptscriptstyle X}^2\}$

 $\underline{p}_{x} := \max\{\underline{p}_{x}^{1}, \underline{p}_{x}^{2}\}, \quad \overline{p}_{x} := \min\{\overline{p}_{x}^{1}, \overline{p}_{x}^{2}\}$

Each approach is discussed in generally the same way:

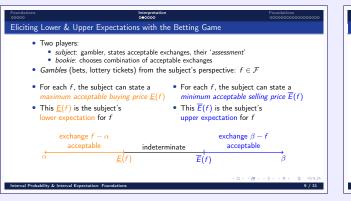
Foundations: basic concepts & axioms

Inference: obtaining values/bounds

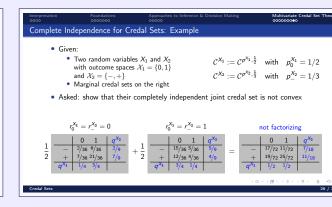
- Interpretation
 - Multivariate models (often)
 - Decision making (often)

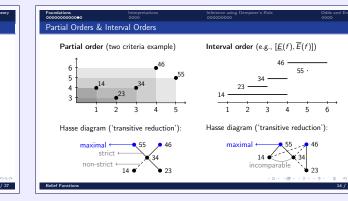
Learning models from data (sometimes)

Some focus area example slides



and predict according to the 'observed' frequency • Event prediction: $\underline{P}_s(S|\mathbf{n}) = \frac{n_S}{n+s}$ and $\overline{P}_s(S|\mathbf{n}) = \frac{n_S+s}{n+s}$ $\underline{\underline{E}}_{s}(f|\mathbf{n}) = \frac{n}{s+n} \underline{E}_{p}(f) + \frac{s}{s+n} \min_{x \in \mathcal{X}} f(x)$ with $p_{x} := \frac{n_{x}}{n}$ Properties of <u>E</u>_s(·|**n**):



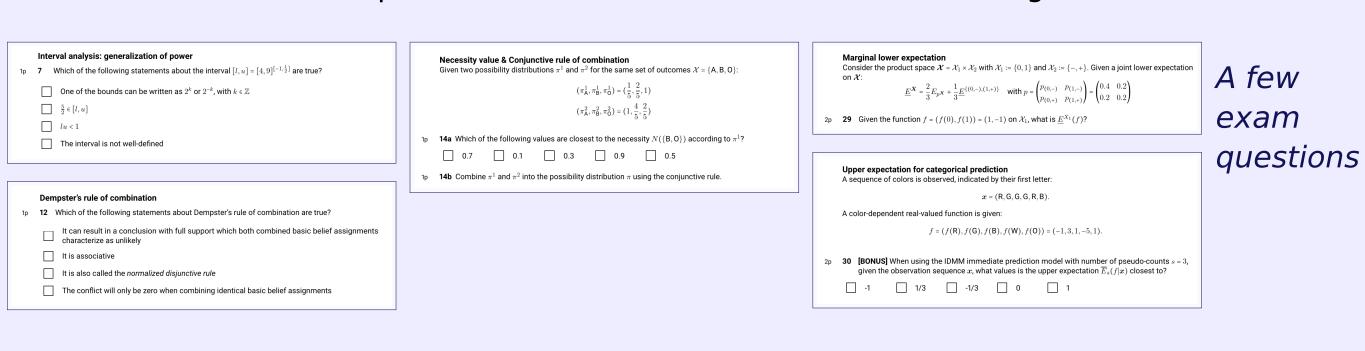


Practice exercises

- On-line quiz per lecture (ungraded, repeatable) Multiple-choice and open questions
- Automated feedback and model answers
- Theory and calculation questions
- Students generally did not participate in a timely manner

Exam

- 30 questions (multiple-choice and open)
- Practice exercises were mostly representative of exam questions
- Level of attainment expected on beforehand was not achieved in general



Problems, Challenges, and Plans

Goal (for us teachers, this time)

Get feedback to improve the course in the coming years

More attention to practice exercises

Problem Exam results showed a lower-than-aimed-for proficiency solving exercises Challenge How do we get students to make the practice exercises in a timely manner? Plan Make the practice exercises a more integrated part:

- Incentivize by making them count for the grade Create time by removing content
- But which content? Create dedicated practice exercise Q&A blocks

Providing more didactic literature

Problem Reports & poster presentations showed that many students encountered difficulties understanding the content of a good deal of the provided literature

Plan Improve the list of provided literature

Challenge Where do we find a sufficiently broad set of didactically written papers?